EFFECTS ON MAXIMUM OVERSHOOT AND SETTLING TIME WHEN BACKLASH IS PRESENT IN A SECOND ORDER SERVOMECHANISM

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Submitted in partial fulfillment of
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MASTER OF SCIENCE
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ABSTRACT

In order to investigate the effects of gear train backlash on the performance of second order servomechanisms, an automatic digital computer was utilized to obtain solutions of numerous second order servo problems. Solutions were obtained for variations of motor and load inertias, motor and load frictions, combined system damping coefficient, and backlash angle, throughout the practical range of these parameters. The data generated by the digital computer solution has been analyzed and presented in graphical form for use in estimating the performance of an arbitrary second order servo system with backlash.

The program was conducted on the Control Data Corporation Model 1604 Computer located at the U. S. Naval Postgraduate School, Monterey, California.

The writers wish to express their appreciation for the assistance and encouragement given them by Professor George J. Thaler of the U. S. Naval Postgraduate School.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Symbol	Description	Units
θ_R	Angular position of the input signal	radians
θ_L	Angular position of the load shaft	radians
θ_M	Angular position of the motor shaft	radians
$\dot{\theta}$	Angular velocity of the motor, gears, and load referred to the load shaft	radians/sec
$\ddot{\theta}$	Angular acceleration of the motor, gears, and load referred to the load shaft	radians/sec/sec
$\dot{\theta}_M$	Angular velocity of the motor referred to the load shaft	radians/sec
$\dot{\theta}_L$	Angular velocity of the load shaft	radians/sec
$\ddot{\theta}_M$	Angular acceleration of the motor referred to the load shaft	radians/sec/sec
$\ddot{\theta}_L$	Angular acceleration of the load	radians/sec/sec
J_T	Equivalent inertia of motor and load referred to the load shaft	slug-feet^2
J_L	Inertia of the load	slug-feet^2
J_m	Inertia of the motor	slug-feet^2
J_M	Inertia of the motor referred to the load shaft	slug-feet^2
F_{total}	Equivalent viscous friction of the motor and load referred to the load shaft	lb-feet/radian/sec
F_T	Load friction plus motor friction referred to load shaft	lb-feet/radian/sec
F_L	Friction constant of the load	lb-feet/radian/sec
F_M	Friction constant of the motor referred to the load shaft	lb-feet/radian/sec
F_m	Friction constant of the motor	lb-feet/radian/sec

K_t	Motor torque constant	lb-feet/ampere
Δ	Backlash angle	radians
ζ	Linear damping coefficient of the closed loop system	none
M_{pt}	Maximum overshoot of load shaft	% of input
ω_n	Undamped natural frequency	radians/sec
ρ	Gear ratio	none
K_g	Motor generator constant	volts/radian/sec
K_e	Error measurement constant	volts/radian
R	Armature resistance	ohms
$K_a = K_t K_e / \rho R$	Armature motor constant	lb-feet/radian
$F_1 = K_t K_g / \rho^2 R$	Armature motor friction constant	lb-feet/radian/sec
$dM_{pt}/d\Delta$	Slope of the overshoot versus backlash curves	% of input/radian
e	Coefficient of restitution	none

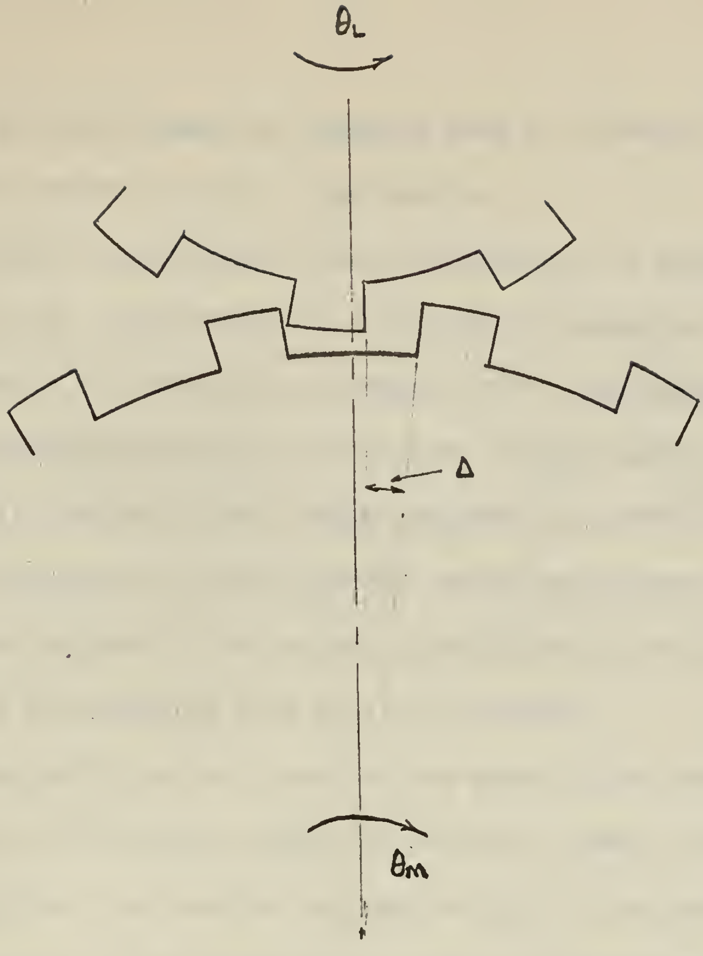
Chapter I

INTRODUCTION

A servomechanism which is designed using linear theory rarely performs as estimated. The discrepancies between design performance and actual performance are caused, in general, by certain non-linearities which are inherent in all systems. Of the many non-linearities which are found in servo systems, the one under investigation here is backlash.

In a servo system, whenever torques are transmitted from a motor to a load through a gear train, there will exist a certain amount of backlash between the gears as shown on figure 1. Present day gear drives cannot transmit loads continuously without some backlash. If the gears are too closely meshed they might bind, wear excessively, or introduce random torque variations appearing as noise. This is particularly true of gear trains which operate over wide temperature variations. Backlash is often controlled so as to introduce a slight dither to a system to provide good lubrication to overcome stictional effects.

It is generally known that backlash in a servo system is destabilizing. However, the estimation of the effect of backlash on the transient response and the ultimate steady state operation of a system presents a very difficult problem. As soon as backlash is introduced into an otherwise linear system, response is then dependent upon the division of inertia between motor and load, the division of viscous friction between motor and load, the damping coefficient of the system when operating as a unit, the amount of energy dissipated as heat when contact is made between the gear teeth, and other phenomena which cannot be predicted. Since the system performance is affected



GEARS ENGAGED

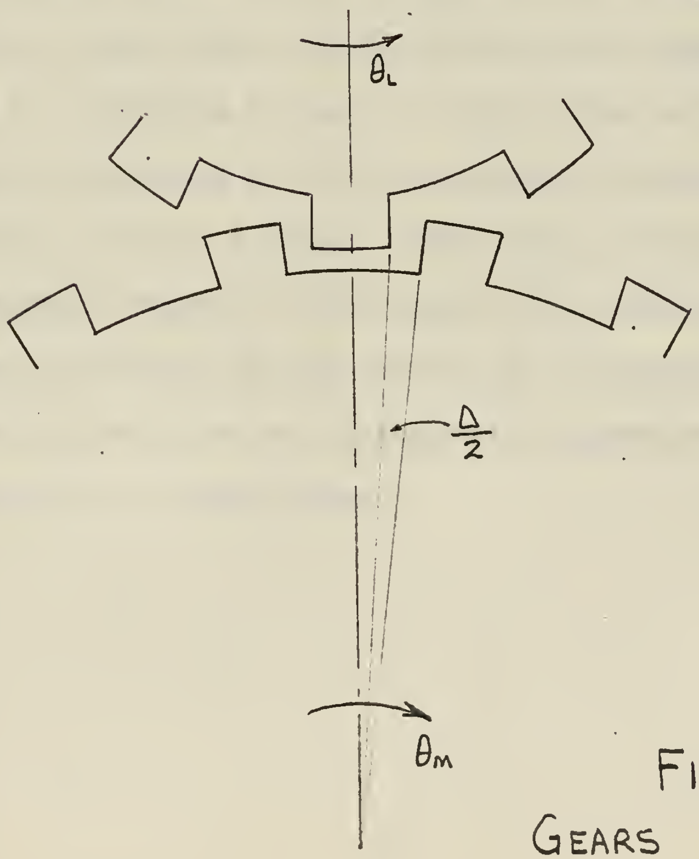


FIG. 1
GEARS DISENGAGED

differently in the presence of backlash when a different set of variables is used, each system presents a new problem.

In practice, knowledge of the performance of a system may be desirable prior to the construction of a mock-up or simulation system. In order to obviate the necessity of estimating the performance by laborious and complicated analytical techniques, design charts are presented in this thesis from which the design engineer can predict the magnitude of maximum overshoot for any arbitrary second order servo with backlash. A qualitative analysis of the effect of backlash on the time of maximum overshoot and the settling time is also presented.

Previous work has been done in this area at the Naval Postgraduate School by W. J. Lutkenhouse, (Ref. 1), and N. C. New, (Ref. 2). Lutkenhouse solved the problem by construction of the phase trajectory on the phase plane. New extended the work of Lutkenhouse by employing an automatic digital computer for the solution of the problem, and was able to collect a large volume of data for use in the construction of design charts for predicting the size of limit cycles in the system. The work done by Lutkenhouse and New was limited to the existence and size of limit cycles in a system. This thesis extends that work to include transient behavior of load motion. The computer program written by New was utilized for the solution of a large number of problems, the data from which was subsequently analyzed and processed for the construction of design curves.

Chapter II

BACKGROUND

Prior to the development of high speed digital computers, it was necessary for the engineer to use long and laborious methods for analyzing the behavior of servomechanism systems containing non-linear elements. Previous investigation of methods used to analyze non-linear systems has resulted in several approaches.

One of the most widely used methods for analyzing the behavior of a non-linearity is the phase plane diagram. At present, this method is limited to control systems represented by a second order differential equation in which the coefficients can be constants or functions of the dependent variable. The solution to this problem is one in which time is not expressed explicitly, but only the velocity and position of the dependent variable for each instant of time are shown. From Ref. 1, it can be noted how tedious, time consuming, and often inaccurate such a method is for obtaining the transient response of a second order servo with some non-linearity.

Another method widely used in non-linear problems is the use of describing functions. This method uses the concept of obtaining an "equivalent" transfer function. If a sinusoidal input of constant amplitude is applied to a non-linear system, the output will contain a number of harmonics of the input frequency. If the fundamental component of the output is assumed to be of the same frequency as the input, then an "equivalent" transfer function can be obtained which is the ratio of the fundamental component of the output to the sinusoidal wave input. Once this relationship of the output to the input of a non-linear system is obtained, it is then used in the same

manner as the transfer function in the linear system. Since this method assumes that the higher harmonics have negligible effect on the system and that the frequency of the input is the same as the output, it is seen that it can only be used for a rough approximation in determining the amplitude and period of the output. This method is also disadvantageous because of the difficulty in the computations associated with the determination of the describing function for a wide range of amplitudes and frequencies. There is no simple method for evaluating the accuracy of a describing function and no good way to check the validity of solutions obtained with it.

With the advent of the analogue computer, another method of calculating system transient performance was available. This method merely simulates the problem of the servo and its non-linearities to obtain the response of the system. This method is of good value in the preliminary evaluation of the system behavior where accuracy is not too important. However, due to the inherent inaccuracies of the analogue computer this method does not give precise results.

By utilizing the digital computer in the solution of non-linear servos, it is possible to overcome many of the disadvantages inherent in the other methods. A non-linear system which can be described by a few accurate or approximate differential equations can be solved on a digital computer resulting in transient and steady state information. In this thesis a digital computer was used to solve many second order servo problems with backlash present. From the wealth of results obtained, the response of any arbitrary second order servo with backlash may be predicted. It is believed that this procedure could be applied to servos of higher order and with other non-linearities.

Chapter III

DISCUSSION OF SYSTEM BEHAVIOR

The system under investigation in this thesis is a simple second order position servo control having the motor coupled to the load through gear trains having backlash. The configuration of the system is illustrated in block diagram form in figure 2. Position feedback is taken from the load shaft including the backlash within the feedback loop. When backlash is outside the feedback loop, i.e., position feedback from the motor shaft, the system is stable and does not result in a limit cycle; however, it is subject to a residual steady state error in response to a step input which error may be as great as the magnitude of the backlash. When included in the feedback loop, backlash is destabilizing and the effect is dependent upon many variables. When a step position input is imposed on the system there is initially a period when the backlash is taken up, and the system performs as a linear system. If the linear system damping coefficient is less than the critical value, the error signal will eventually cause the motor shaft to reverse its direction of rotation. A separation between the motor and the load will occur in the cycle of motion whenever the deceleration of the motor becomes greater than that of the load. However, if the frictional forces on the load produce a deceleration greater than the maximum deceleration of the motor, the motor and the load will maintain contact during this period and become separated only when the motor velocity reverses. The precise instant at which motor and load separate depends upon the nature of the frictional forces acting and the division of inertia between the motor and load. When the gears do break contact, the load drifts and the system is no longer a closed loop system but is

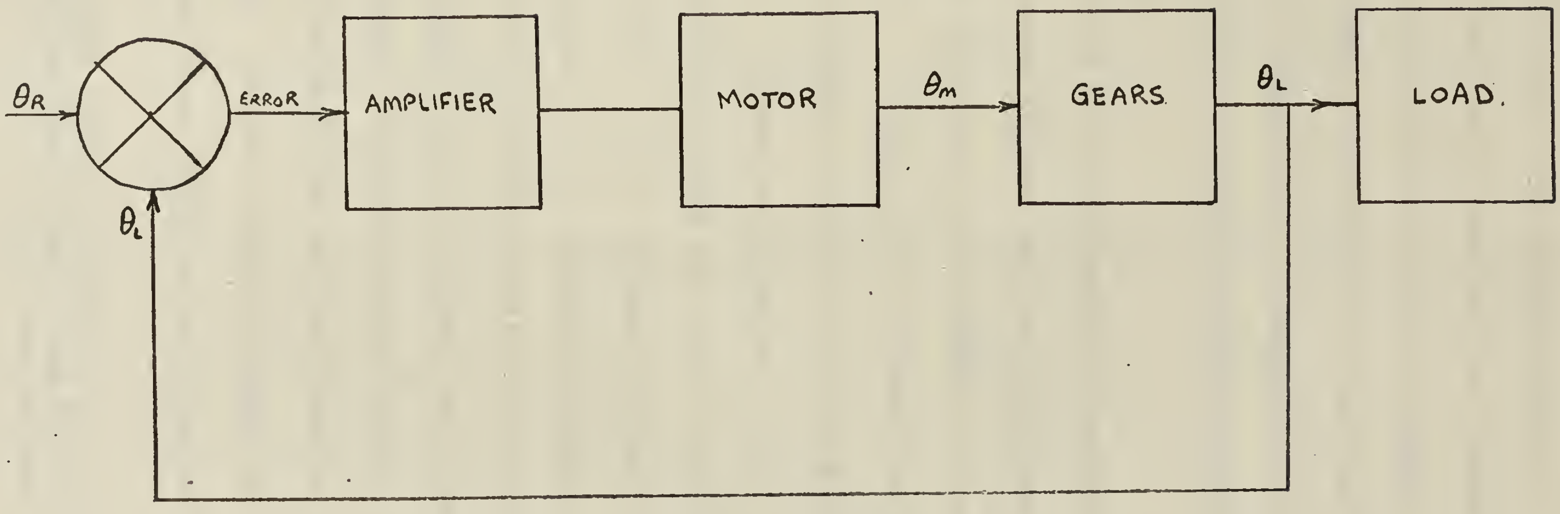


FIG. 2 SECOND ORDER SERVOMECHANISM WITH BACKLASH BETWEEN MOTOR AND LOAD

driven open loop. Whenever the total amount of backlash is taken up and gear contact is made, the action which follows is very complex. The basic laws which govern the ensuing motion are the law of conservation of energy and the law of conservation of momentum. In this thesis the assumption is made that the contact is a plastic one with no gear bounce, and that energy is dissipated as heat due to the plastic contact. Thus only the law of conservation of momentum are applied.[1] Following the gear contact phase, the system is again combined, and the cycle of events begins again.

The motion of the motor and load is illustrated in figures 3 and 4. Angular velocity versus angular position is plotted and the phase trajectories for both motor and load are shown for two conditions of inertia and friction ratios. In figure 3 it is noted that at point A, the system reacts to a step input of 1 radian by moving in a trajectory in accordance with the differential equation for the linear system. At point B the gears which couple the motor to the load separate; the dotted trajectory from B to C' showing the path followed by the motor and the solid line from B to C showing the path of the drifting load. The gears separate whenever the velocity of the load exceeds that of the motor, which depends upon the inertia and friction distribution between motor and load. In this case the inertia of the motor is equal to the inertia of the load, but the load friction is much greater than the motor friction. For this reason the gears do not separate until the motor has sufficient error signal in the opposite direction to cause the

[1] A study is being carried on concurrently to include the conservation of energy and the elastic impact case by Lt. N. O. Anderson and Lt. T. W. Luckett at the U. S. Naval Postgraduate School.

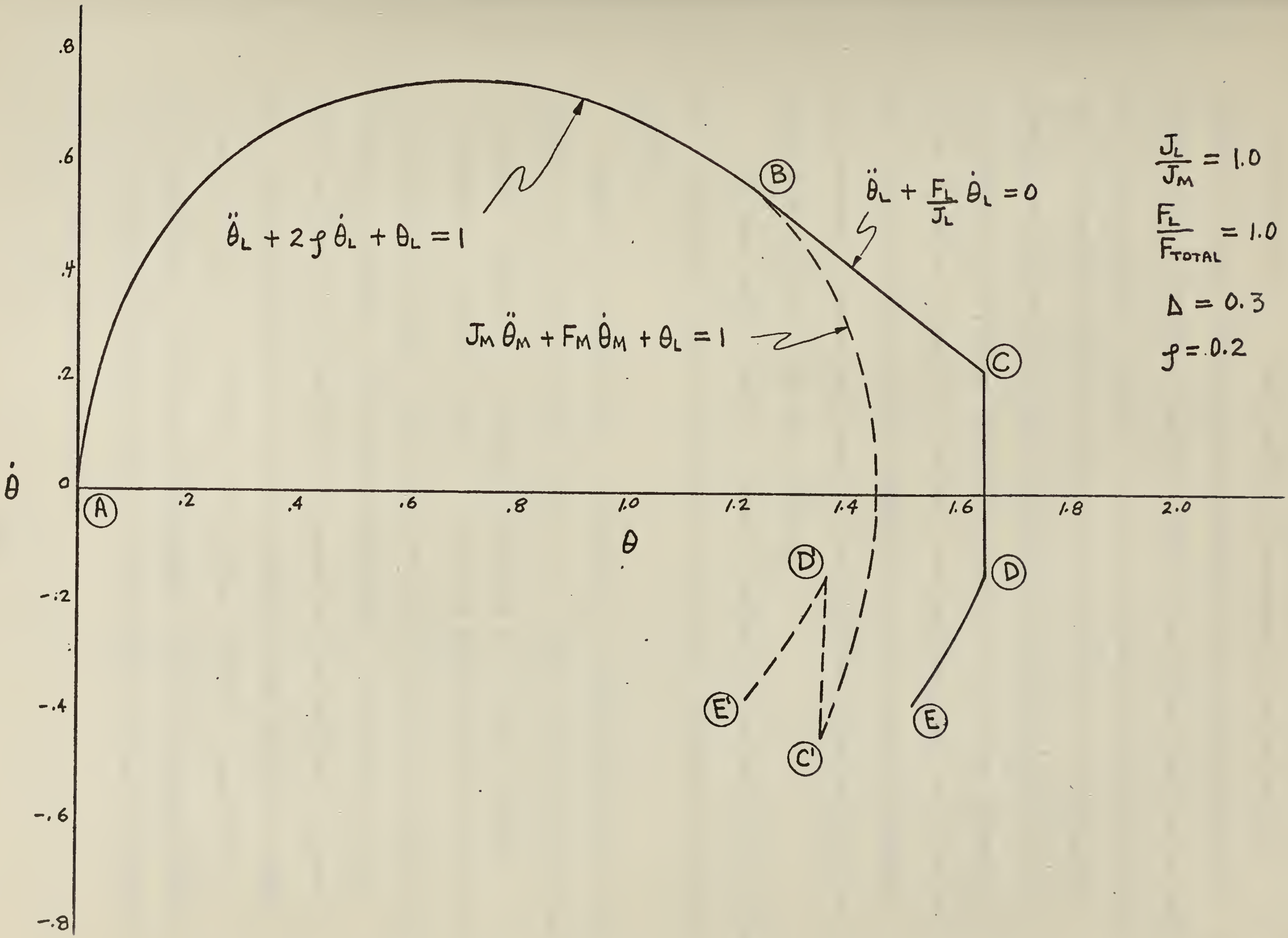


FIG. 3 PHASE TRAJECTORY FOR SHOWING BACKLASH EFFECTS (EXAMPLE 1)

motor to decelerate faster than the load. As the error signal (which is determined by the load position) increases, the motor velocity decreases to zero and begins accelerating in the opposite direction. When the load and motor positions differ by an amount equal to the backlash angle as shown at point C and C', the gears make contact and the law of momentum is applied causing the motor and load to immediately assume the same velocity, that velocity being dependent upon the relative masses, or inertias, of the motor and load. This new velocity is shown at points D and D'. The motor and load now both move in accordance with the combined motion equation but in the opposite direction and separated by an amount equal to the magnitude of the backlash, as shown from D' to E' and from D to E.

The same cycle of events occurs throughout the motion of the system. The effect on the first overshoot is observed from this much of the trajectory.

Figure 4 shows the effect of backlash on a system which has a load inertia greater than the motor inertia and a motor friction greater than the load friction, with the same backlash as in the first example. It is important to notice at point B that the gears separate earlier than in the previous example. Both the heavier load inertia and lighter load friction contribute to its slower deceleration. As before, when the motor and load are separated by the amount of the backlash the gears make contact and the motor and load assume the same velocity as shown by point D and D'. Note that the combined velocity is positive in this case because of the high load momentum. Thus the combined motion is in the positive direction with the error signal causing the motor to act as a brake on the system motion.

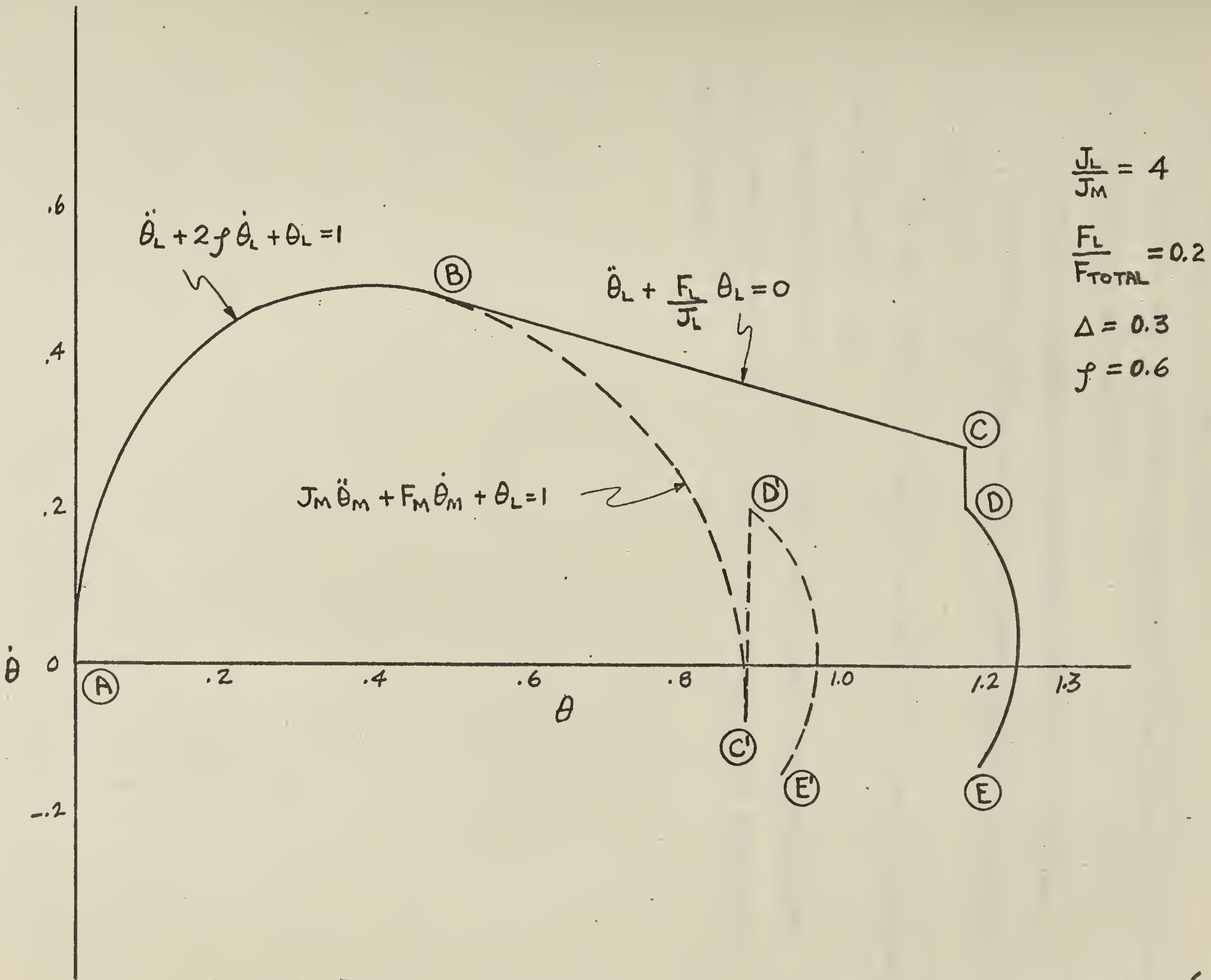


FIG. 4 PHASE TRAJECTORY FOR SHOWING BACKLASH EFFECTS (EXAMPLE 2)

These two cases serve to indicate the fact that every case must be studied individually since every combination of inertias and frictions cause considerably different results.

In order to make the problem mathematically tractible, the following assumptions have been made:

1. Backlash is the only non-linearity present in the system.

2. Backlash is present only in the gears coupling the motor to the load.

3. The gear contact is plastic thus applying the law of conservation of momentum only.

It is believed that the results obtained with these assumptions give a good approximation of the effects of backlash on a second order servo system.

Chapter IV

SOLUTION OF EQUATIONS OF MOTION

When operating as a combined system the servo motion can be described by the differential equation,

$$\ddot{\theta}_L + 2\zeta\omega_n\dot{\theta}_L + \omega_n^2\theta_L = \omega_n^2\theta_R \qquad \text{(IV-1)}$$

When the load drifts free within the backlash region the load motion follows the differential equation,

$$\ddot{\theta}_L + \frac{F_L}{J_L}\dot{\theta}_L = 0 \qquad \text{(IV-2)}$$

and the equation for the motion of the motor is,

$$J_M\ddot{\theta}_M + F_M\dot{\theta}_M + \theta_L = 1 \qquad \text{(IV-3)}$$

The characteristics of the second order servo with backlash is independent of the undamped natural frequency. The proof is developed in Ref. 2. Therefore, all of the problems solved and used for the construction of the design charts are based on an undamped natural frequency of one radian per second. The input for all problems was a step input of one radian. Thus equation (1) becomes,

$$\ddot{\theta}_L + 2\zeta\dot{\theta}_L + \theta_L = 1 \qquad \text{(IV-1}^1\text{)}$$

The differential equations IV-1^1, IV-2, and IV-3, for the second order servo with backlash were solved on the Control Data Corporation Model 1604 Computer. The program used was based on the Runge-Kutta Gill numerical integration method of solving differential equations. A complete description of the program and an explanation of the solution are given in Ref. 2. The program is included in Appendix C. By merely insert-

ing into specific memory locations the system parameters for the problem to be solved, the solution could be obtained for any desired second order servo.

A study of the describing differential equations reveals that there are five system parameters which are independent. These are J_M, F_M, J_L, F_L, and Δ . To facilitate the graphing of the results, ratios of $\frac{J_L}{J_M}$ and $\frac{F_L}{F_{total}}$ were selected, and from these selected ratios discrete values of the parameters J_L, F_L, J_M, and F_M, were calculated for use in the computer solution.

The system damping coefficient is determined by the relationship,

$$2\zeta = \frac{F_{TOTAL}}{J_T}$$

Taking $J_T = J_L + J_M = 1$ simplified calculations, but did not destroy the generality. The system problem was solved for all combinations of the following ranges of inertia and friction ratios and damping coefficient:

$$\frac{J_L}{J_M} = 1/50,\ 1/9,\ 1/4,\ 1,\ 4,\ 9, \text{ and } 90$$

$$\frac{F_L}{F_{total}} = 0.1,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0$$

$$\zeta = 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.8,\ 1.0$$

The manner in which the results are presented in the final design charts permits interpolation through the practical range of system parameters.

The amount of backlash, Δ , used in the majority of the problems was 0.3 radian and it was a purpose of this thesis to determine the

effect of other magnitudes of backlash. At the outset several problems were solved varying the amount of backlash while holding the other parameters at constant values. Figures 5-7 are plots of the results of these solutions. Two items of significant interest were noted from these results. First, the amplitude of maximum overshoot varies linearly with the magnitude of the backlash angle. This fact is significant since it permitted the use of a constant backlash angle in the solution of the problems, yet, in effect, produced results for any amount of backlash. Secondly, the amplitude of maximum overshoot at zero backlash angle agrees with that obtained by the analytical solution of the second order linear system. This agreement is taken as a verification of the accuracy of the computer solutions.

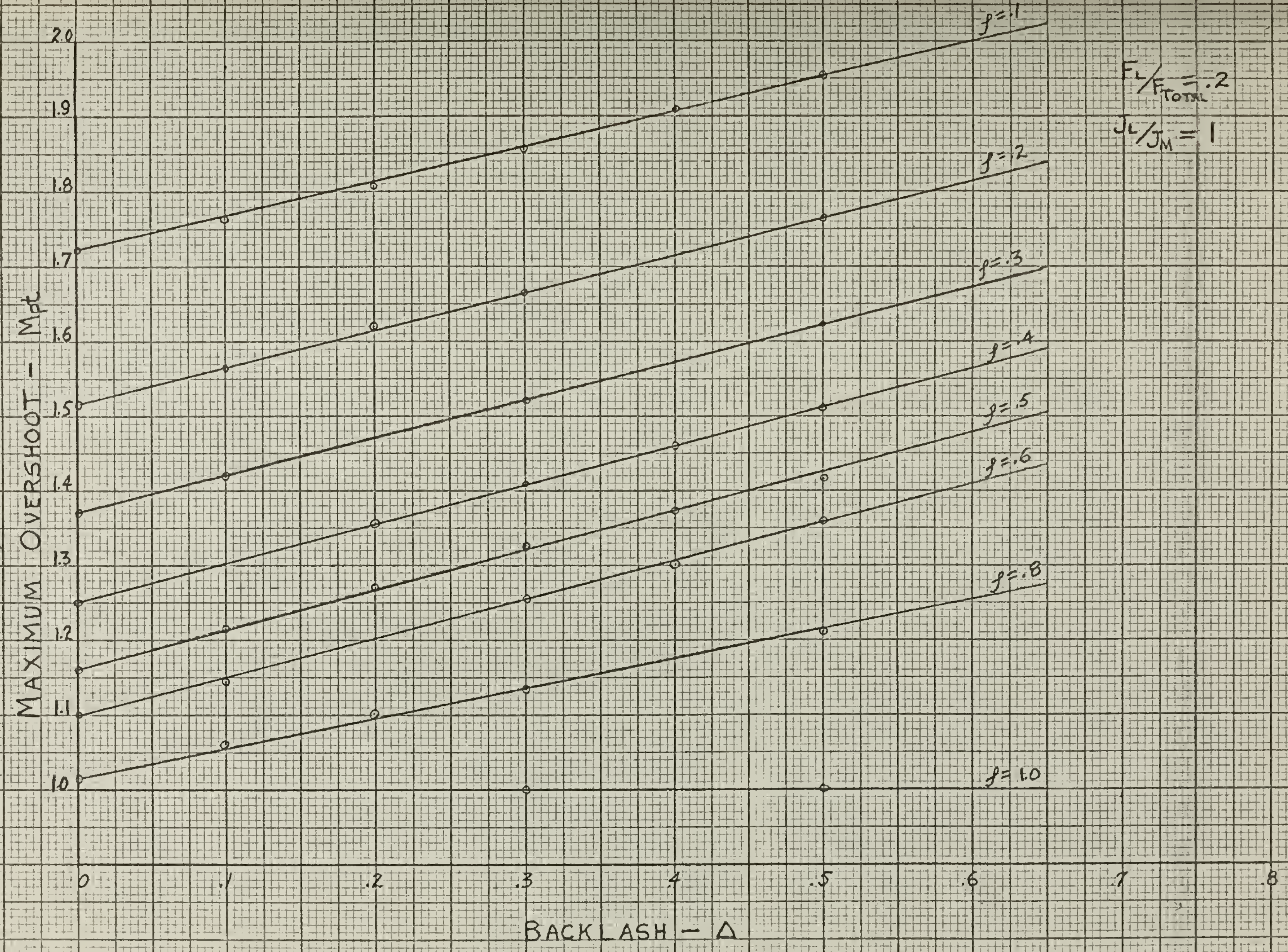


FIG. 5 MAXIMUM OVERSHOOT VS. BACKLASH FOR FRICTION RATIO OF .2 AND J_L/J_M OF 1.0

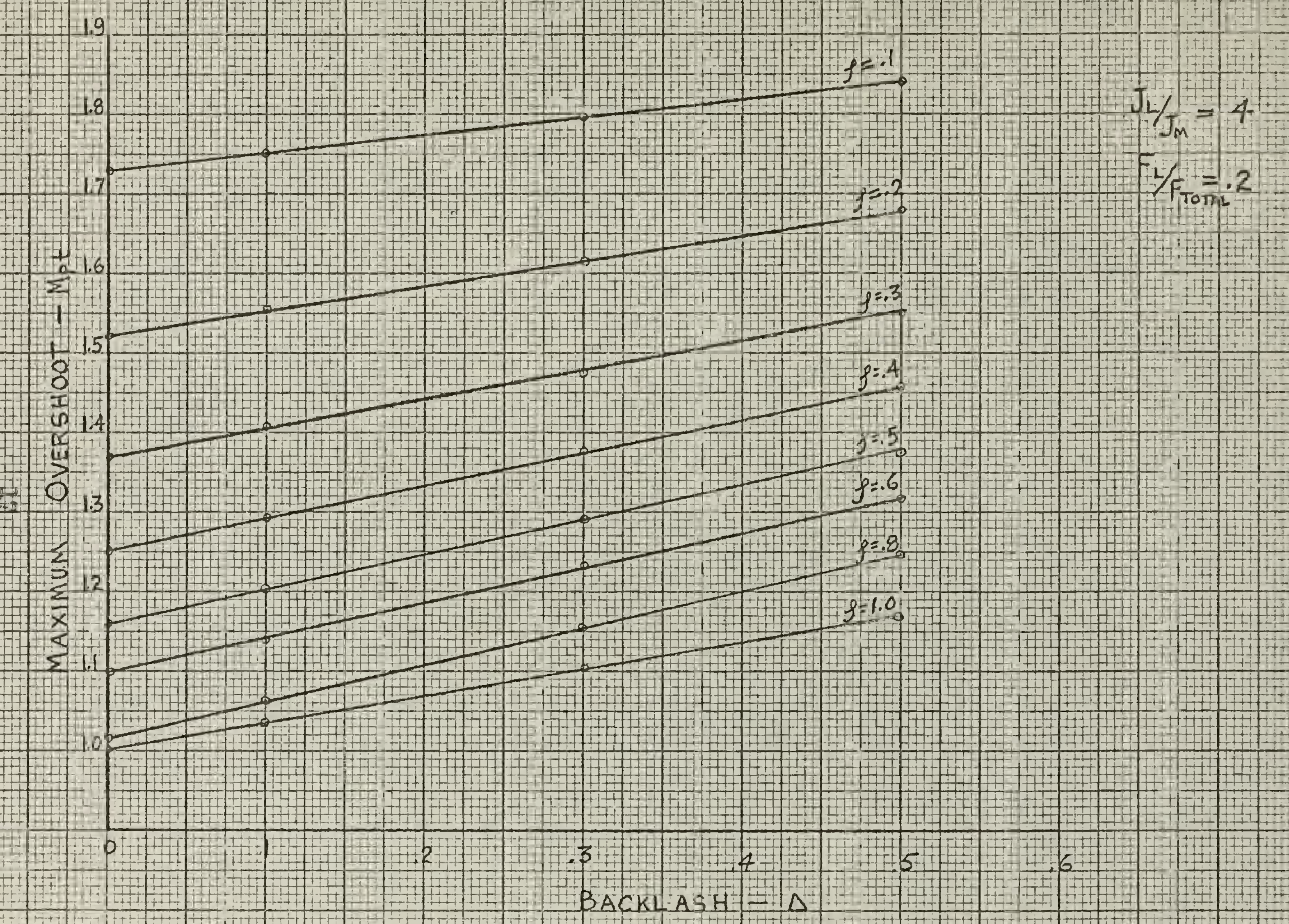


FIG. 6 MAXIMUM OVERSHOOT VS. BACKLASH FOR FRICTION RATIO OF .2 AND $J_L/J_M = 4$

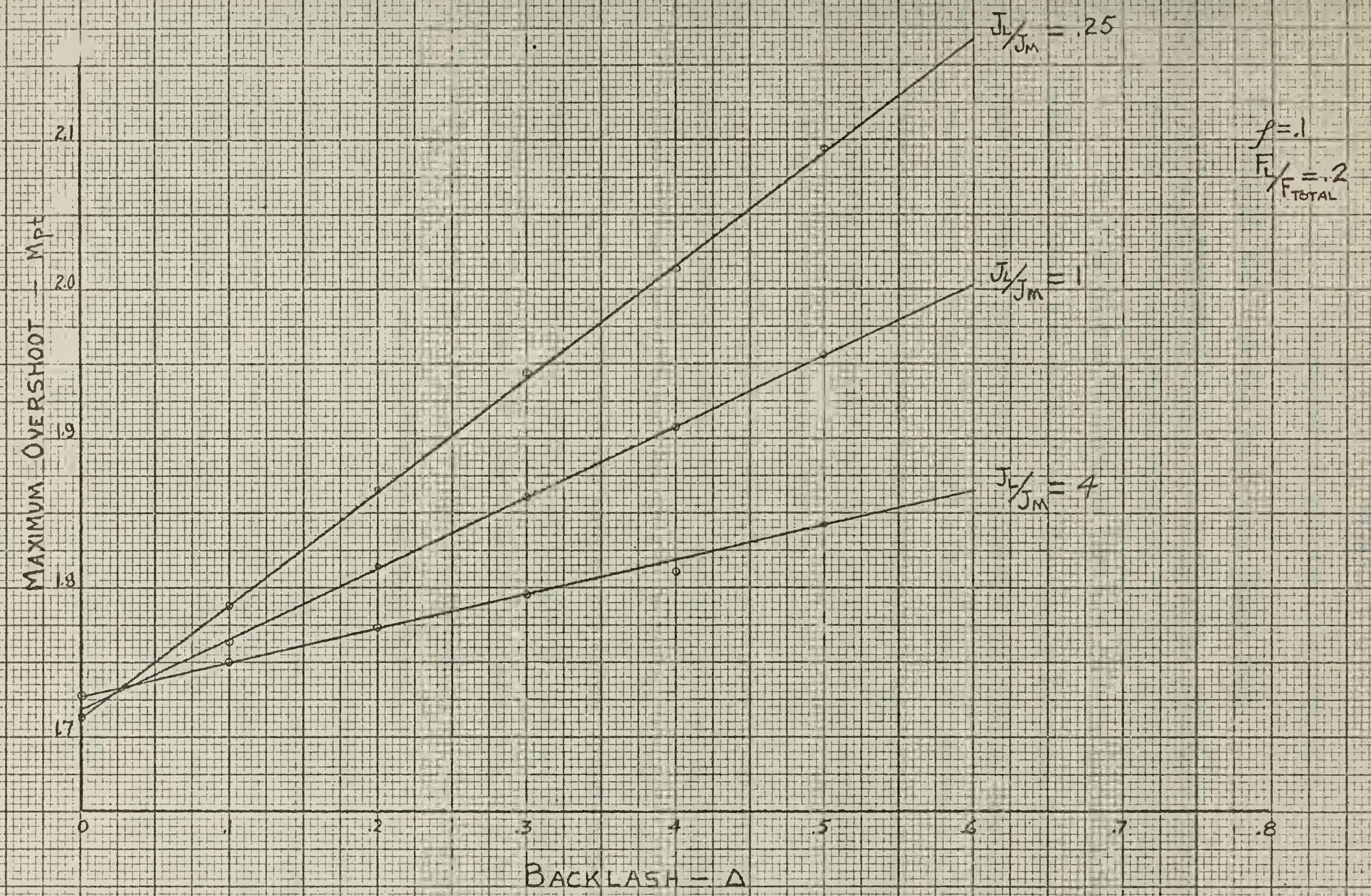


FIG. 7 MAXIMUM OVERSHOOT VS. BACKLASH FOR FRICTION RATIO OF .2 AND DAMPING COEFFICIENT OF .1

Chapter V

DISCUSSION OF RESPONSE CHARACTERISTICS

A. Maximum Overshoot for the Plastic Impact Case.

After numerous data had been accumulated, several schemes of presenting the information were attempted to find an optimum method for use. One such attempt is shown in figure 8 which is a plot of maximum overshoot versus the ratio of load friction to the total friction at a constant damping coefficient. This method was found to have serious disadvantages. Because of appreciable crossing of the lines of constant inertia ratio, it was found difficult to interpolate throughout the desired range of inertia ratios. Also there was generally a small variation of maximum overshoot with friction ratios.

Another method which was attempted was a plot of maximum overshoot versus damping coefficient at constant friction and inertia ratios. These curves showed the effect of various inertia and friction ratios in a system with backlash on the linear system curve of overshoot versus damping coefficient (figure 9), but was found difficult to use. The number of curves needed to present the data would have been excessive. Also interpolation throughout the desired range of inertia ratios would have been extremely difficult.

The curves of figures 10 through 15, which are plots of maximum overshoot versus J_L/J_M, were found to be the most desirable method of presenting the data. None of the disadvantages apparent in the other methods is present here. From these six charts an accurate prediction of the maximum overshoot can be made for any arbitrary second order servo system with backlash using the following equation:

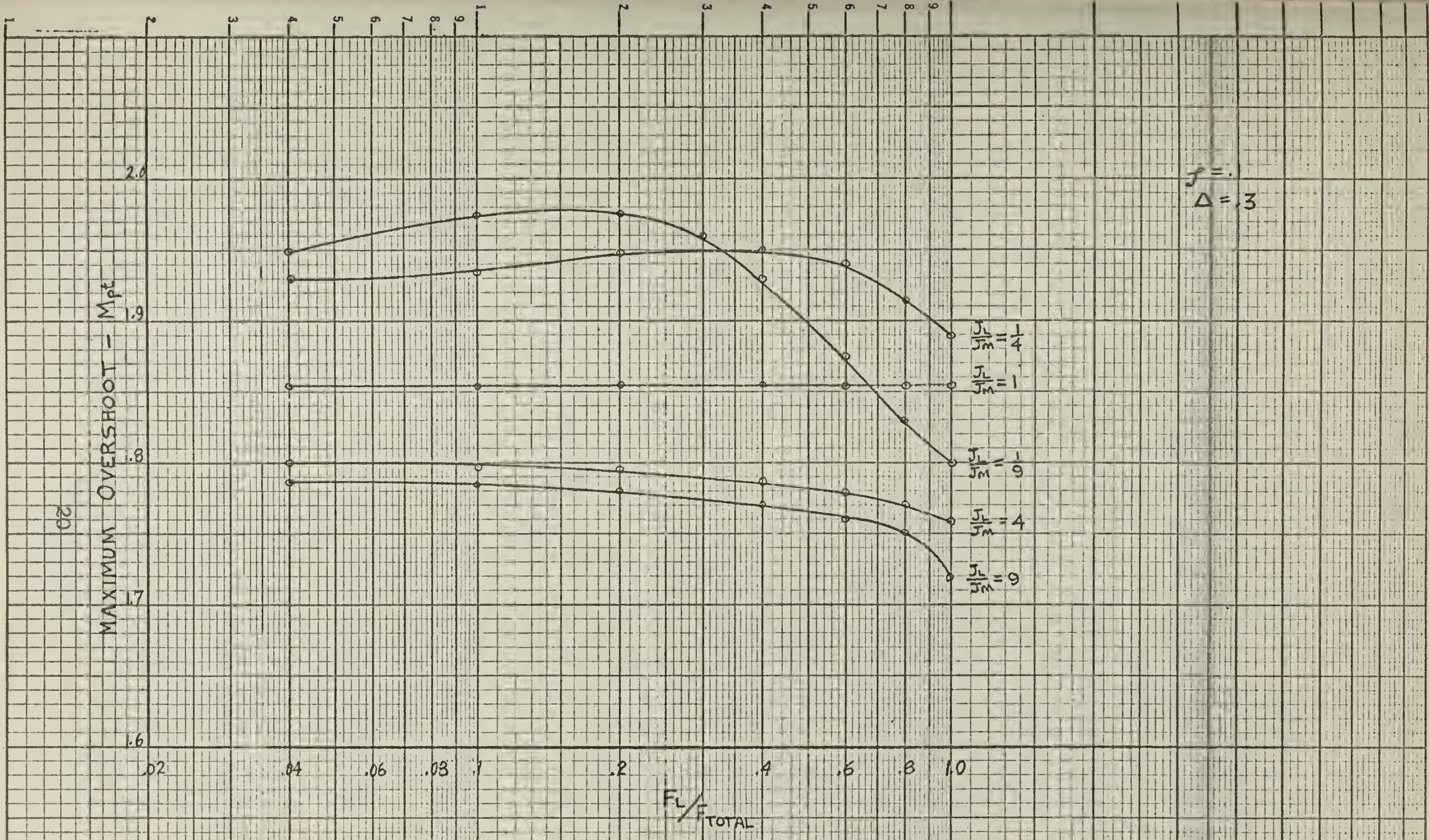


FIG. 8 MAXIMUM OVERSHOOT VS. F_L/F_{TOTAL} FOR DAMPING COEFFICIENT OF .1 AND BACKLASH OF .3

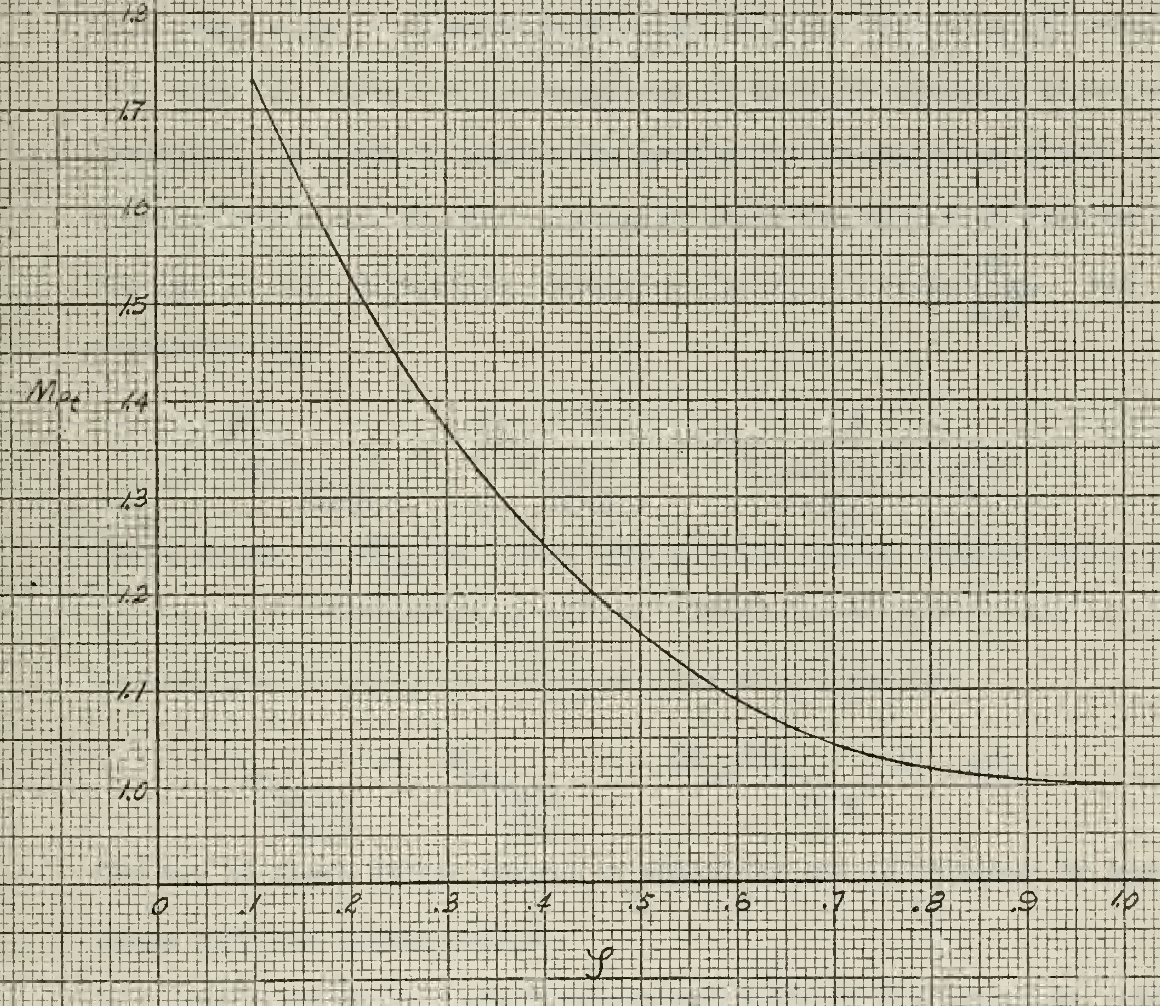


FIG. 9 SECOND ORDER SERVO RESPONSE CHARACTERISTICS
LINEAR CASE

$$M_{pt}\Big|_{\substack{\Delta=\Delta_1\\ f=f_1}} = M_{pt}\Big|_{\substack{\Delta=0\\ f=f_1}} + \Delta_1/0.3\left[M_{pt}\Big|_{\substack{\Delta=.3\\ f=f_1}} - M_{pt}\Big|_{\substack{\Delta=0\\ f=f_1}}\right] \quad (V\text{-}1)$$

The use of these charts in predicting system response is explained and illustrated in Appendix A.

As was pointed out earlier, the solutions were made and the results were plotted using a backlash angle of 0.3 radian. Thus the values of amplitude of maximum overshoot on the curves are significant only in that they represent the slope of the overshoot versus backlash straight lines. This slope must be calculated in order to scale the value of overshoot read from the curve for the magnitude of backlash actually expected. Therefore, in order to refine the prediction scheme, other design curves, figures 16 through 21, were calculated using the curves of figures 10 through 15 just discussed. Along the curves the slope $\frac{dM_{pt}}{d\Delta}$ was calculated by,

$$\frac{dM_{pt}}{d\Delta}\Big|_{f=f_1} = \left[M_{pt}\Big|_{\substack{\Delta=.3\\ f=f_1}} - M_{pt}\Big|_{\substack{\Delta=0\\ f=f_1}}\right] \div .3 \quad (V\text{-}2)$$

These values of $dM_{pt}/d\Delta$ were then plotted versus inertia ratios at constant friction ratios as before. These curves give a more straight-forward prediction method with the following equation:

$$M_{pt}\Big|_{\substack{\Delta=\Delta_1\\ f=f_1}} = M_{pt}\Big|_{\substack{\Delta=0\\ f=f_1}} + dM_{pt}/d\Delta\Big|_{f=f_1} \times \Delta_1 \quad (V\text{-}3)$$

The use of these design charts is explained and illustrated in Appendix A along with the former method.

Figure 23 shows a graphical presentation of the transient response of the system containing various parameters. A comparison of curves (3) and (4) readily shows the effect of backlash on the maximum overshoot with other parameters remaining constant. A comparison of curves (1)

and (4) shows the effect of varying the damping ratio of the system with other parameters remaining constant.

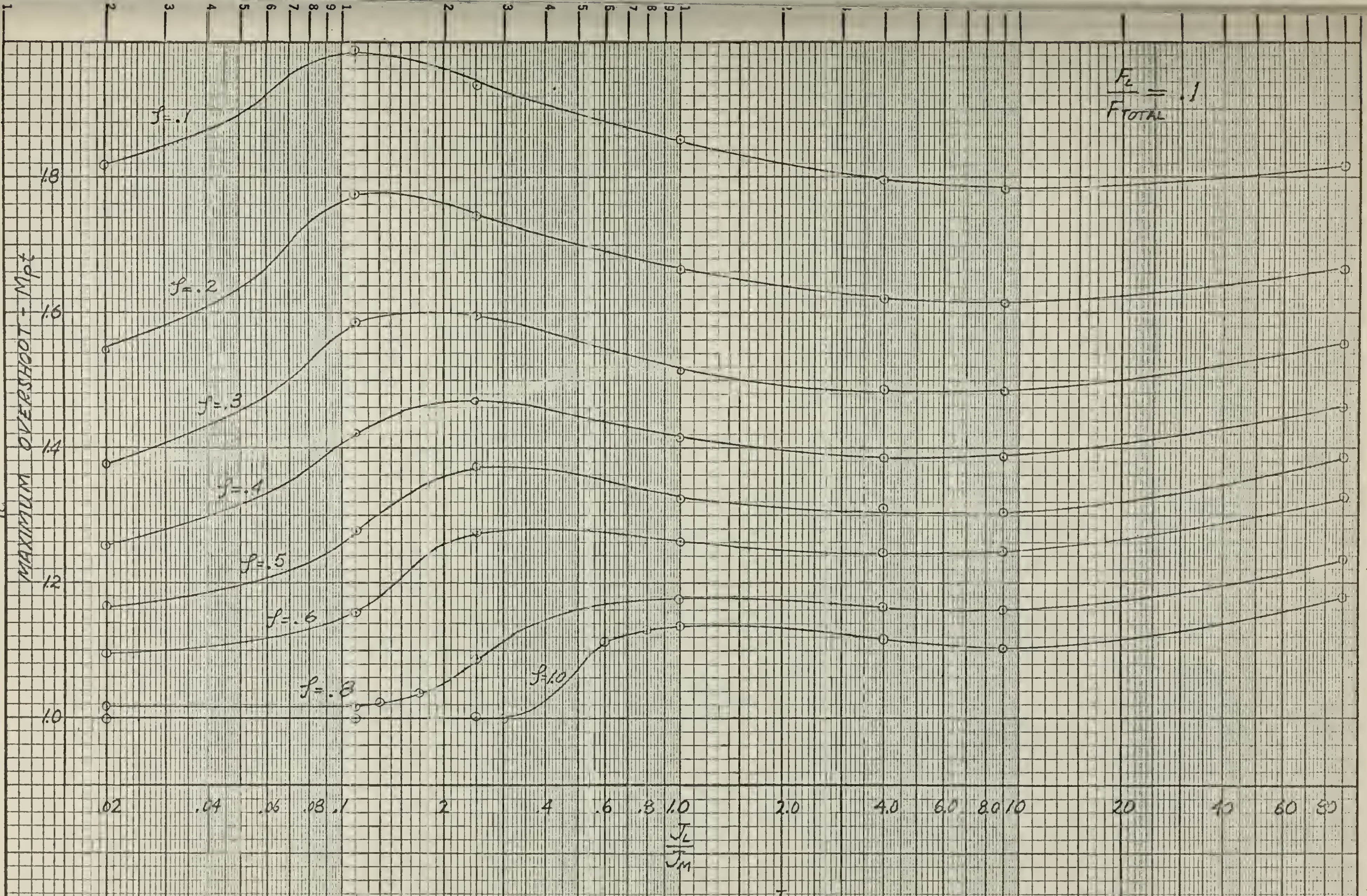


FIG. 10 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 0.1 AND BACKLASH OF 0.3

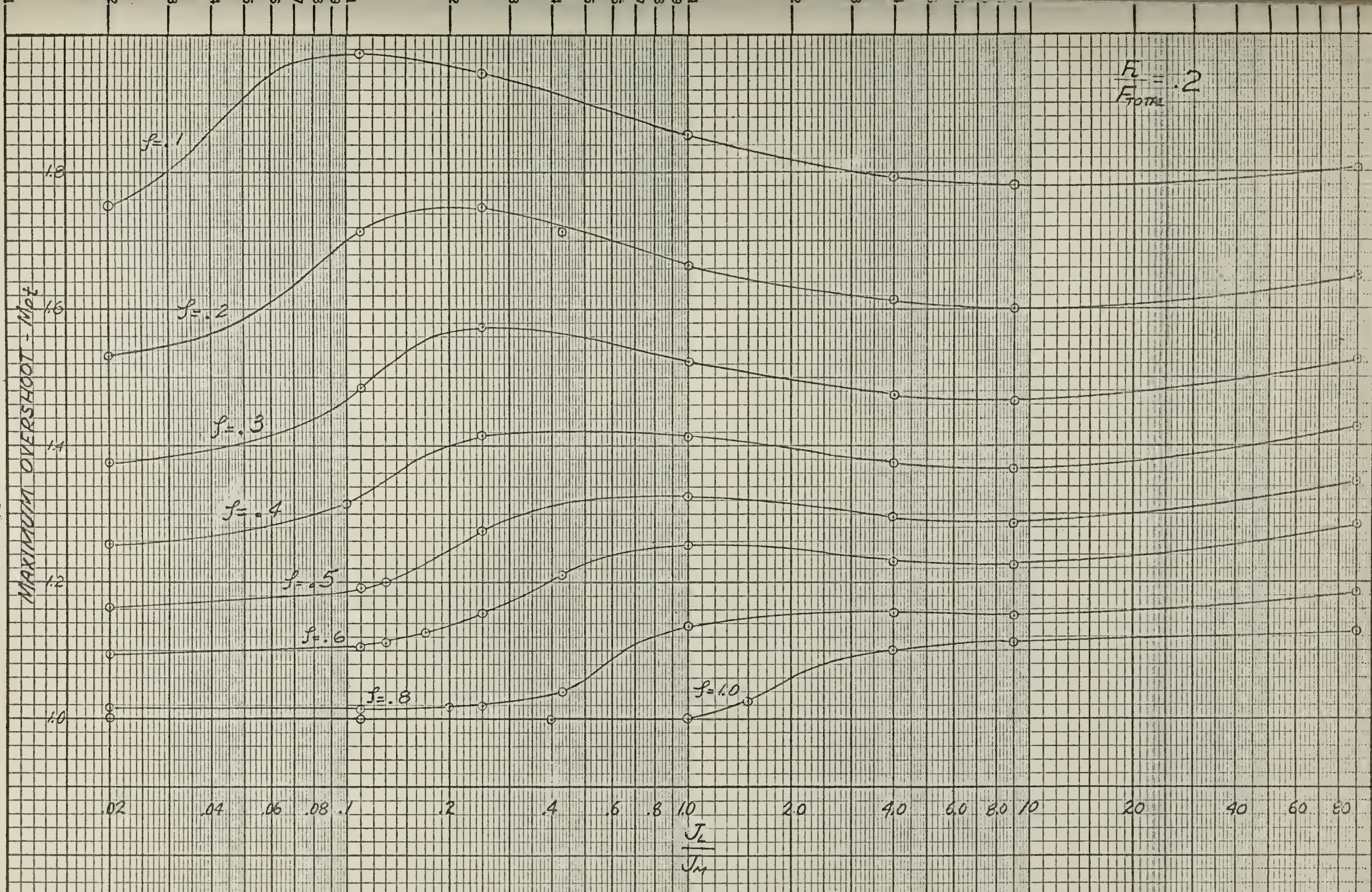


FIG. 11 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 0.2 AND BACKLASH OF 0.3

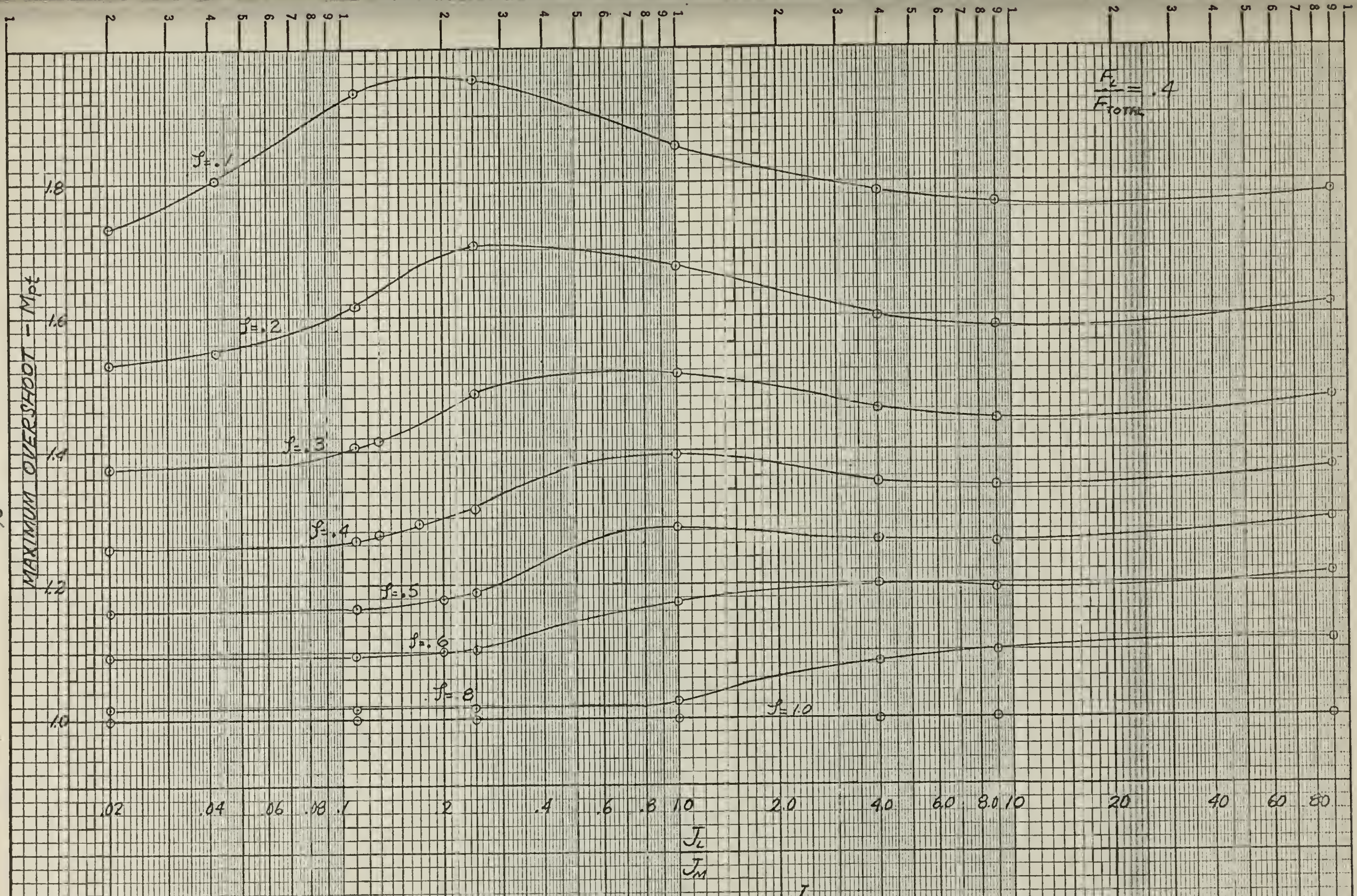


FIG. 12 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 0.4 AND BACKLASH OF 0.3

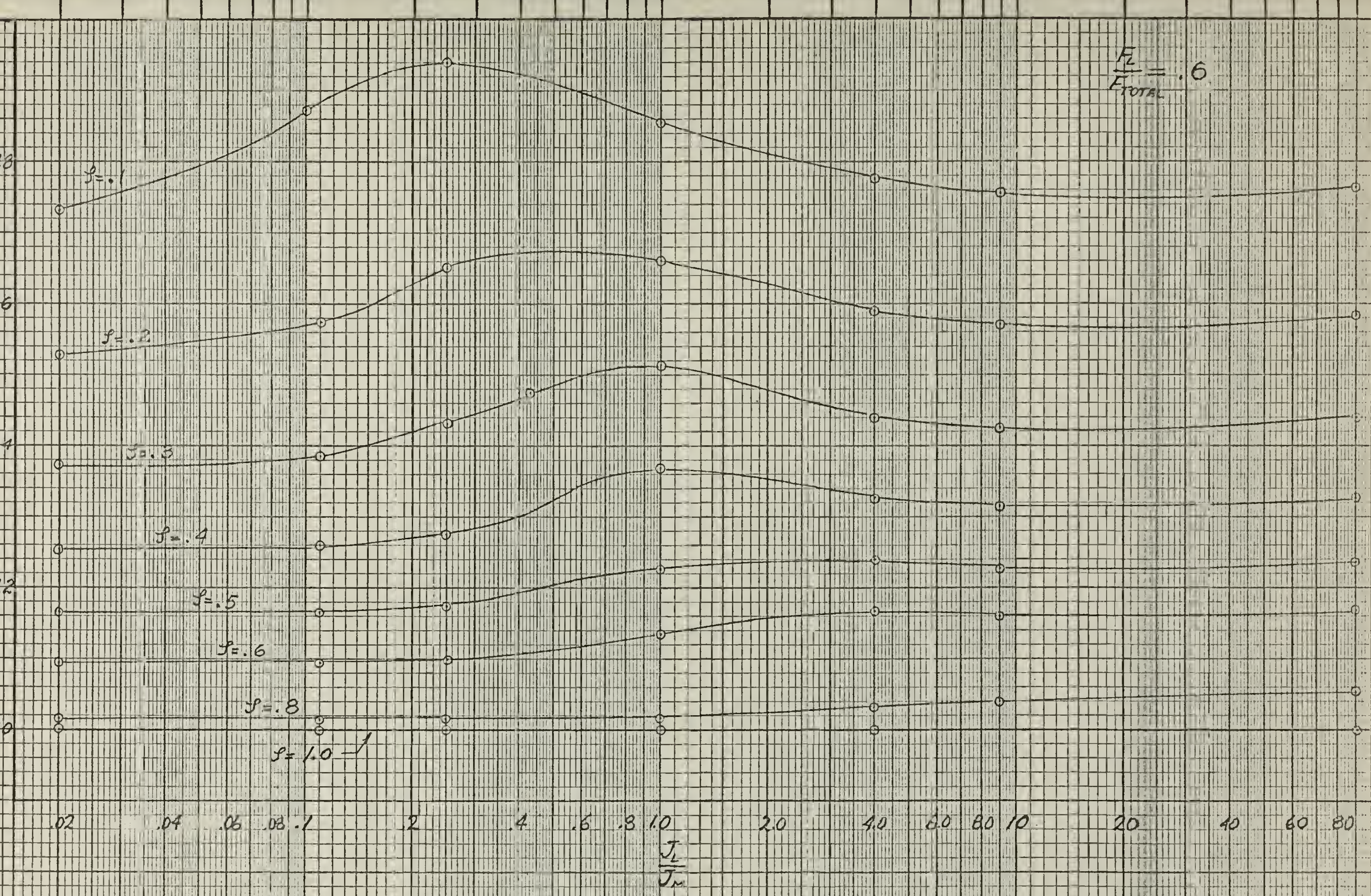


FIG 13 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 0.6 AND BACKLASH OF 0.3

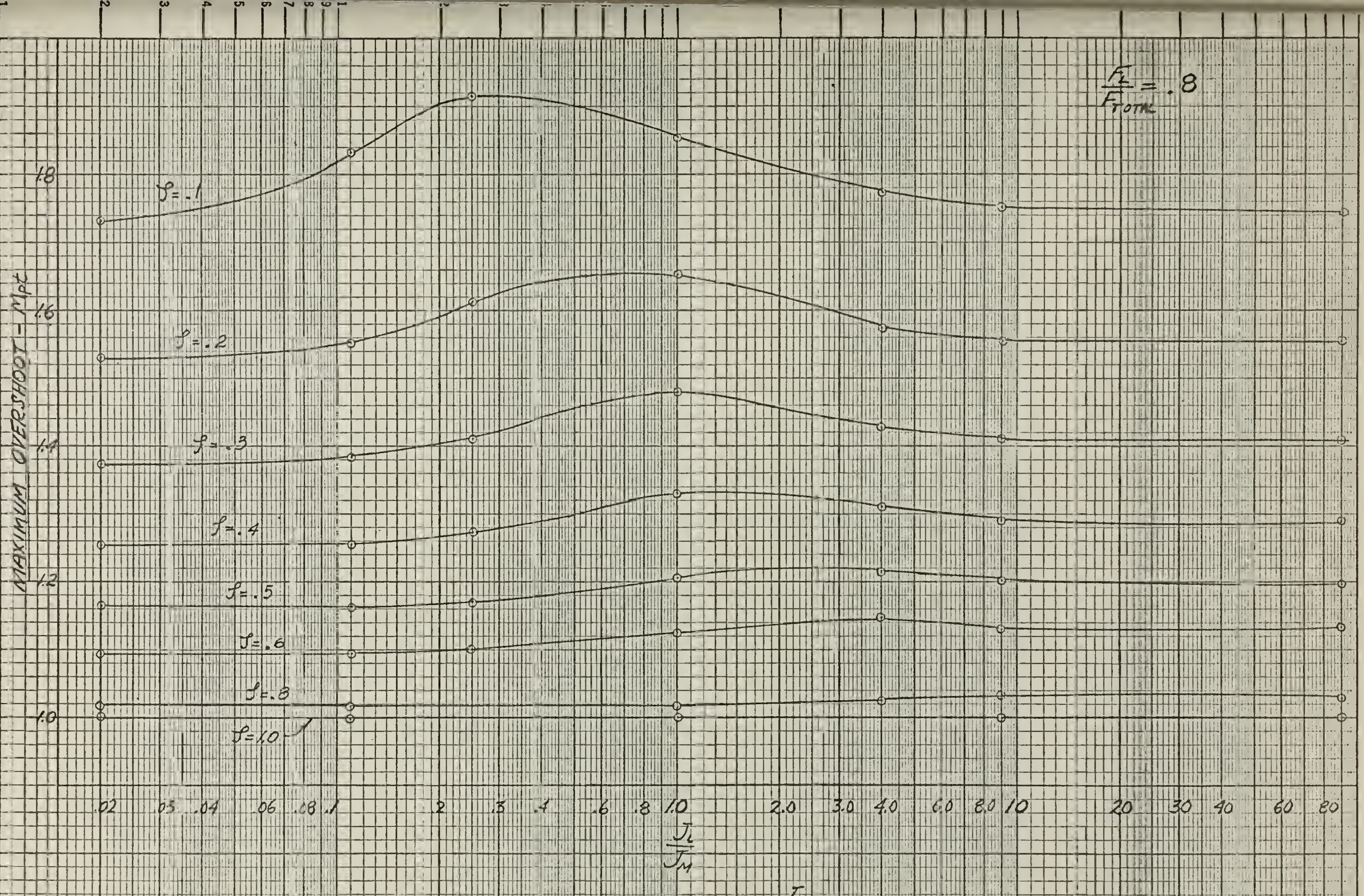


FIG. 14 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 0.8 AND BACKLASH OF 0.3

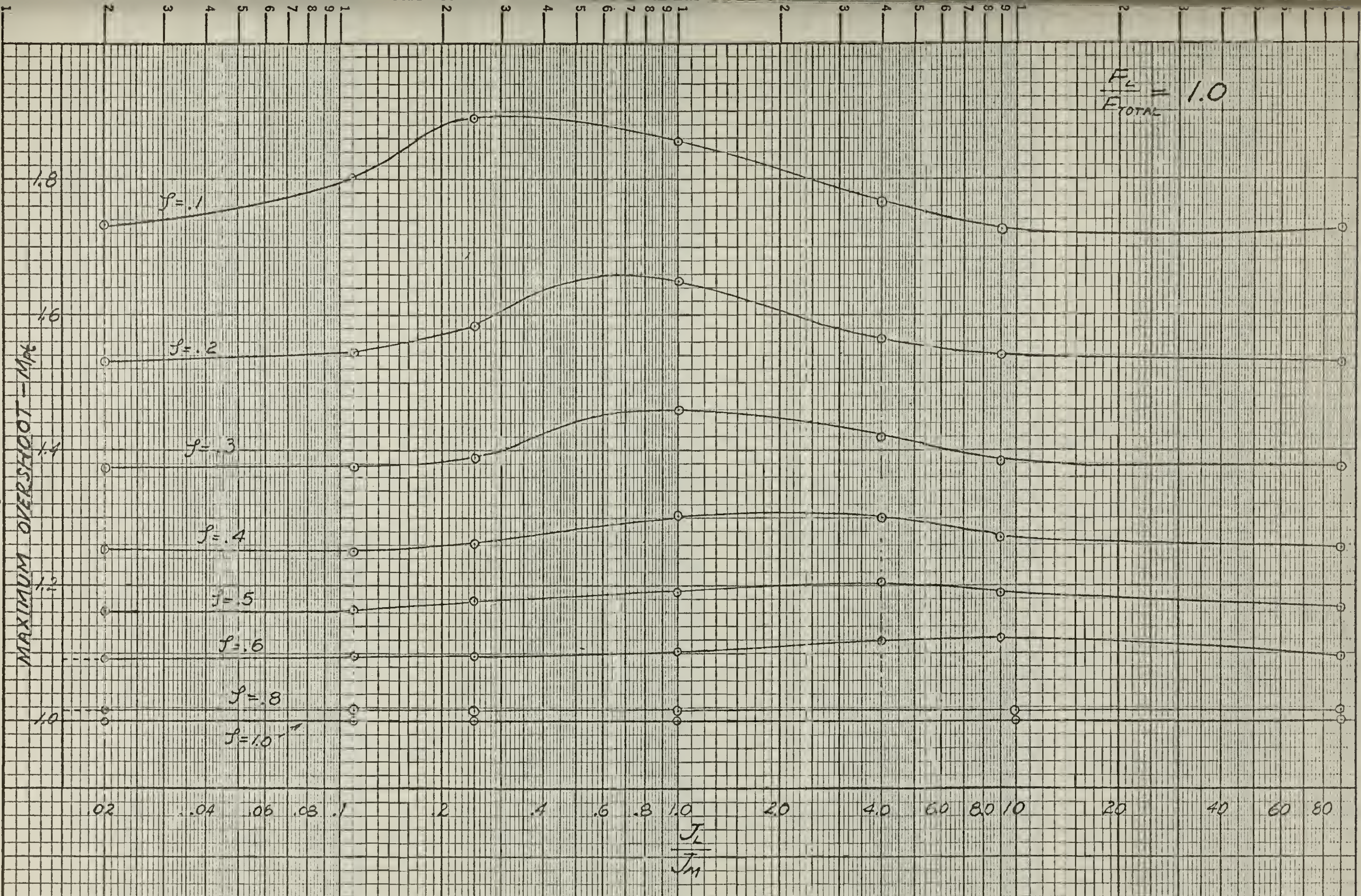


FIG 15 MAXIMUM OVERSHOOT VS. $\frac{J_L}{J_M}$ FOR FRICTION RATIO OF 1.0 AND BACKLASH OF 0.3

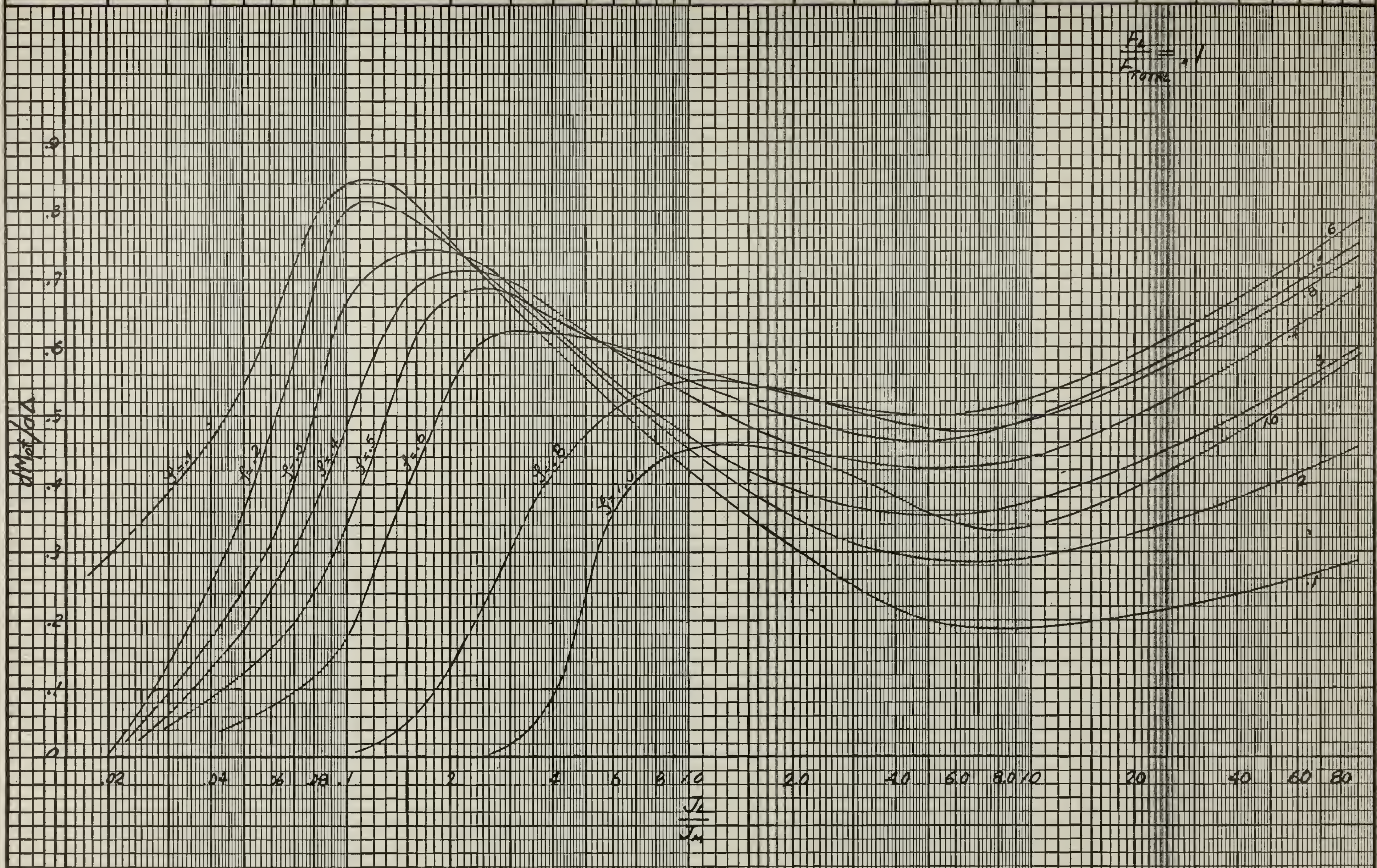


FIG. 16 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = .1\right)$

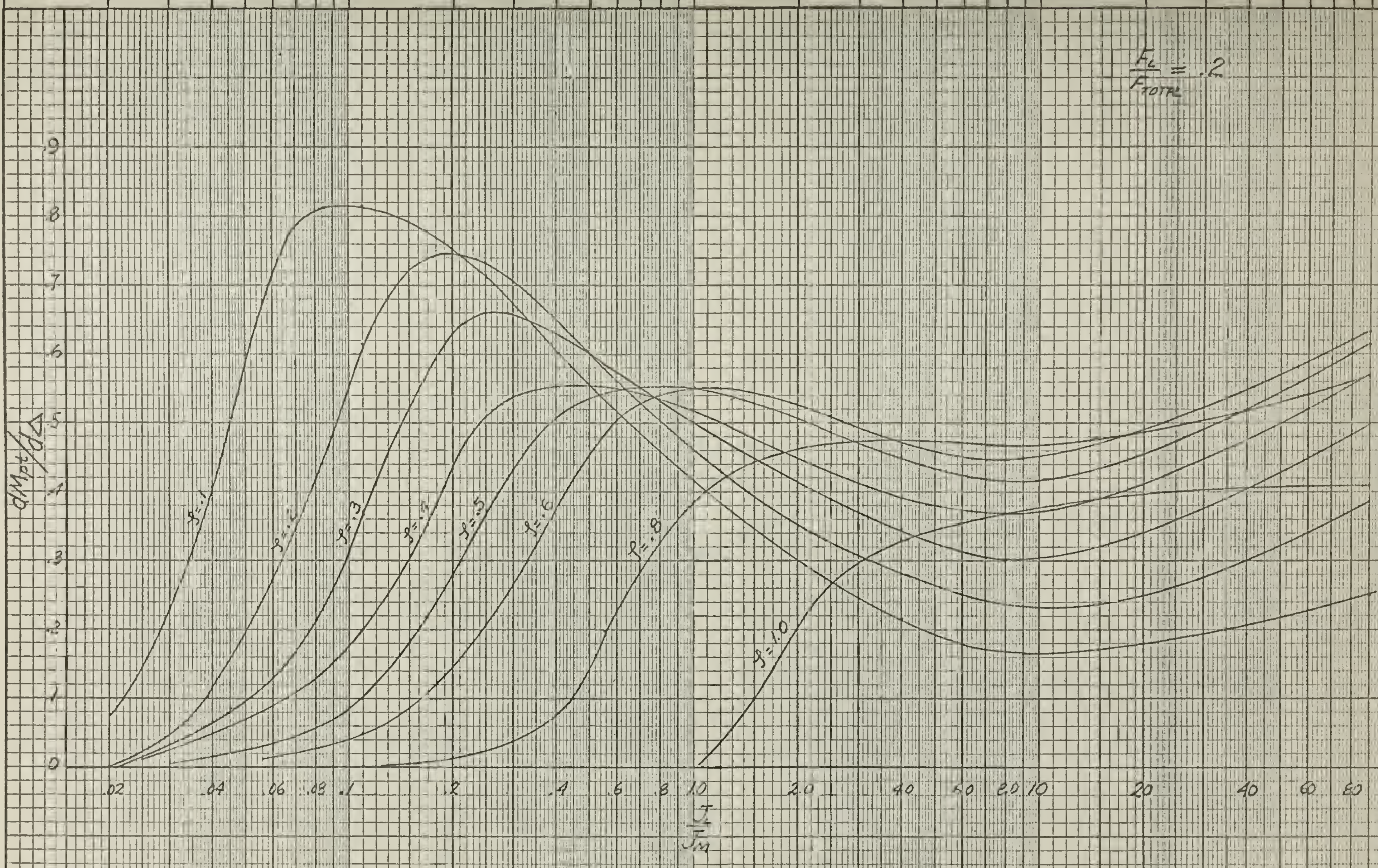


FIG. 17 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = .2\right)$

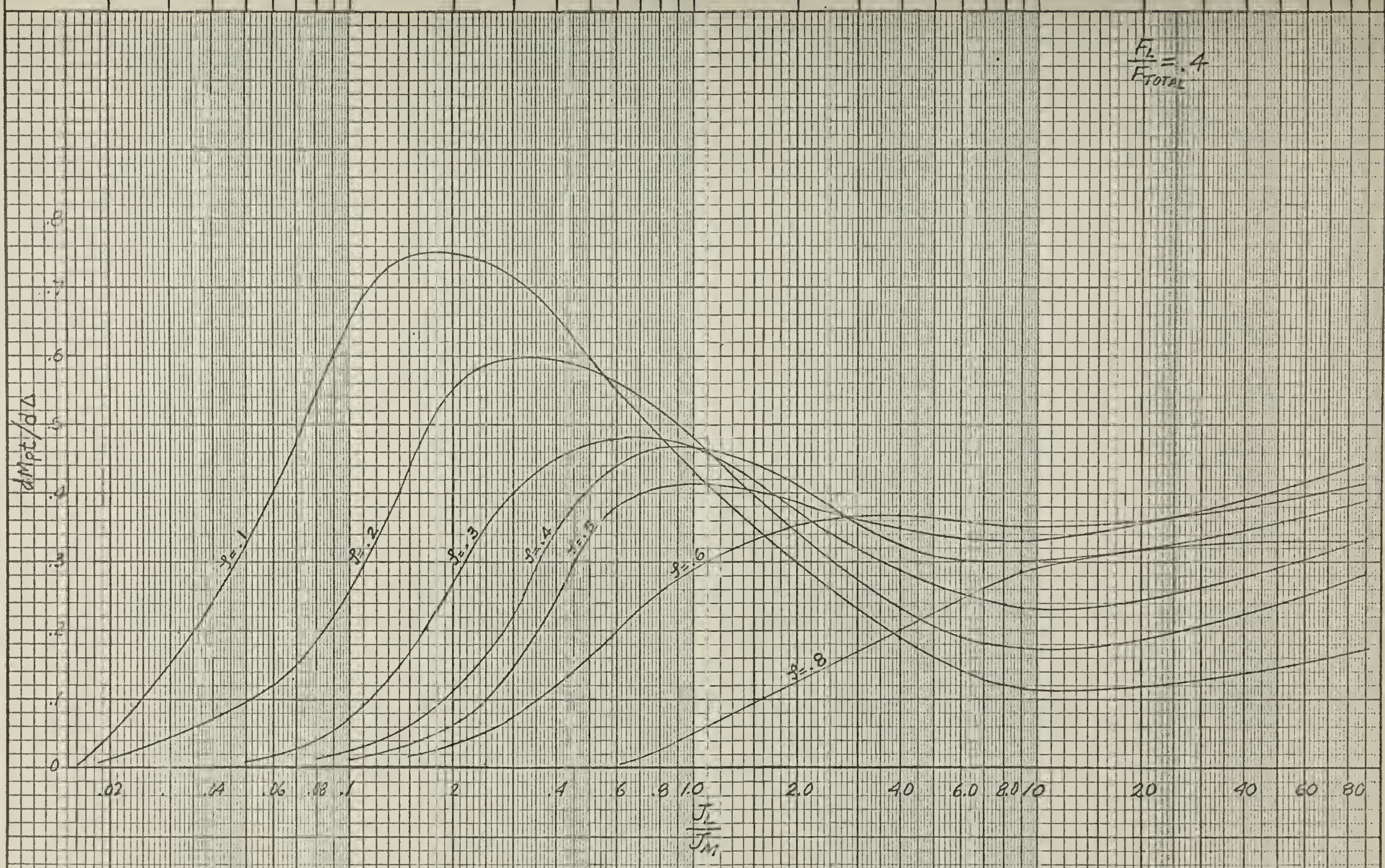


FIG. 18 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = .4\right)$

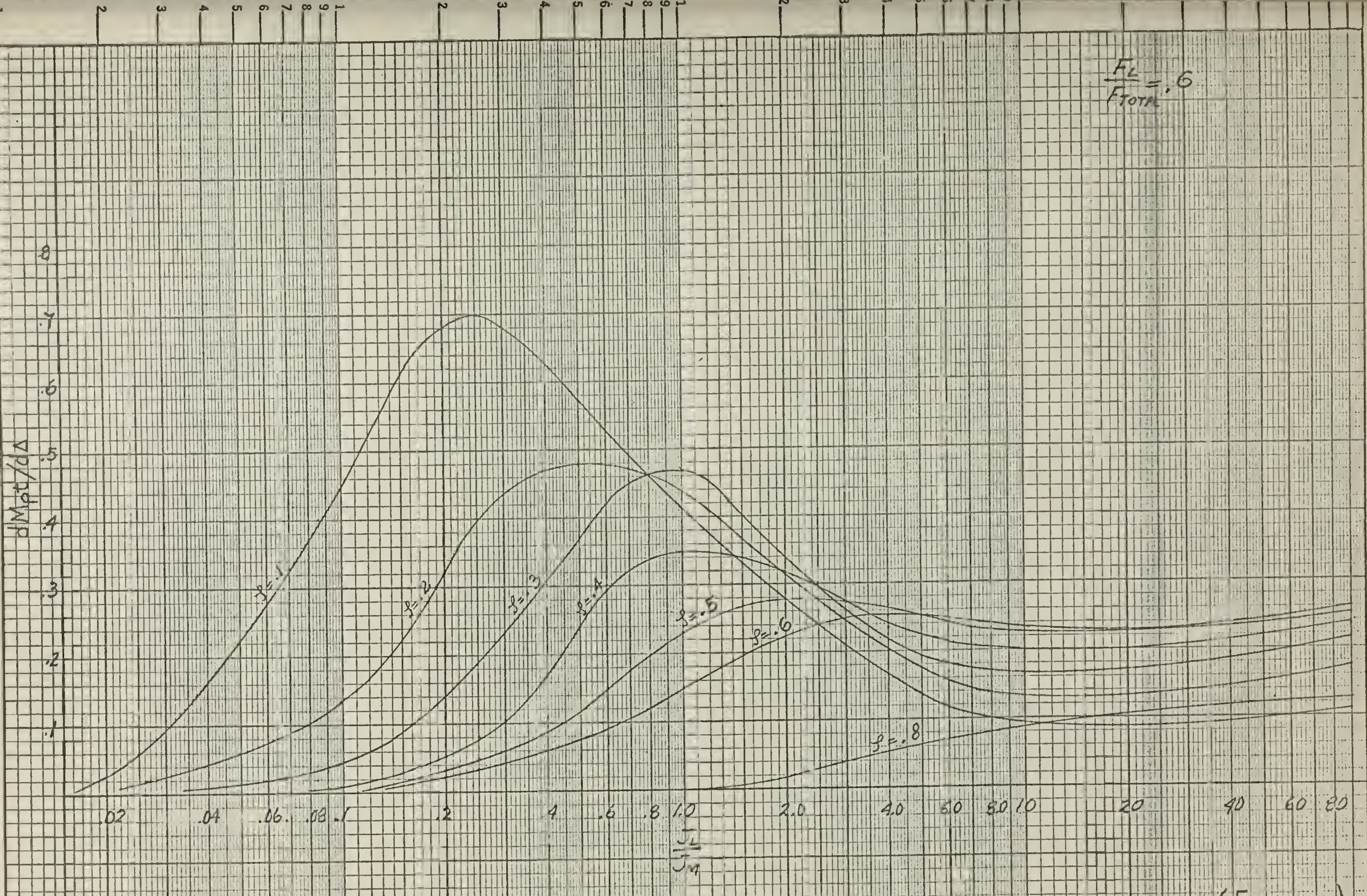


FIG 19 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = .6\right)$

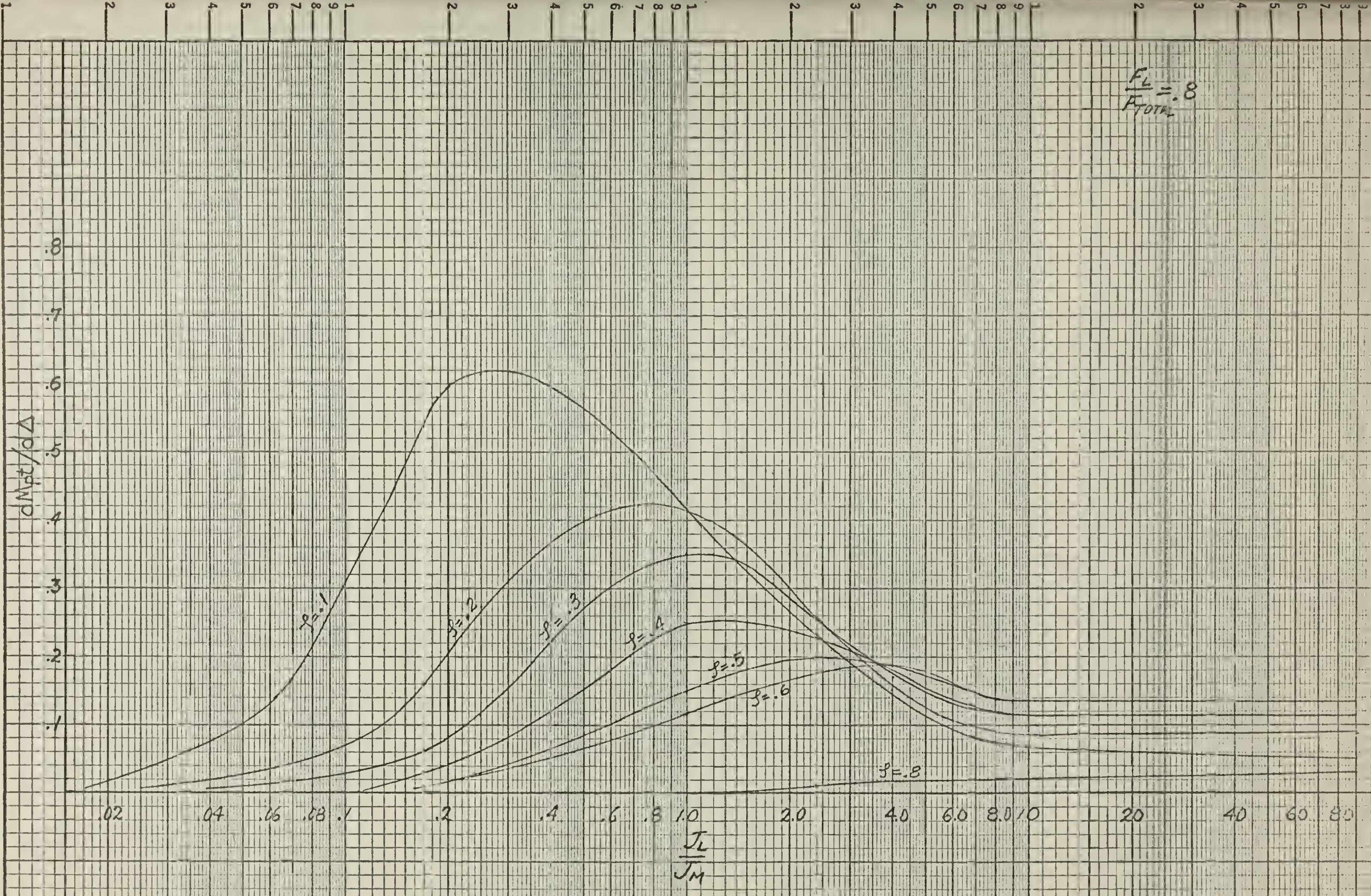


FIG. 20 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = .8\right)$

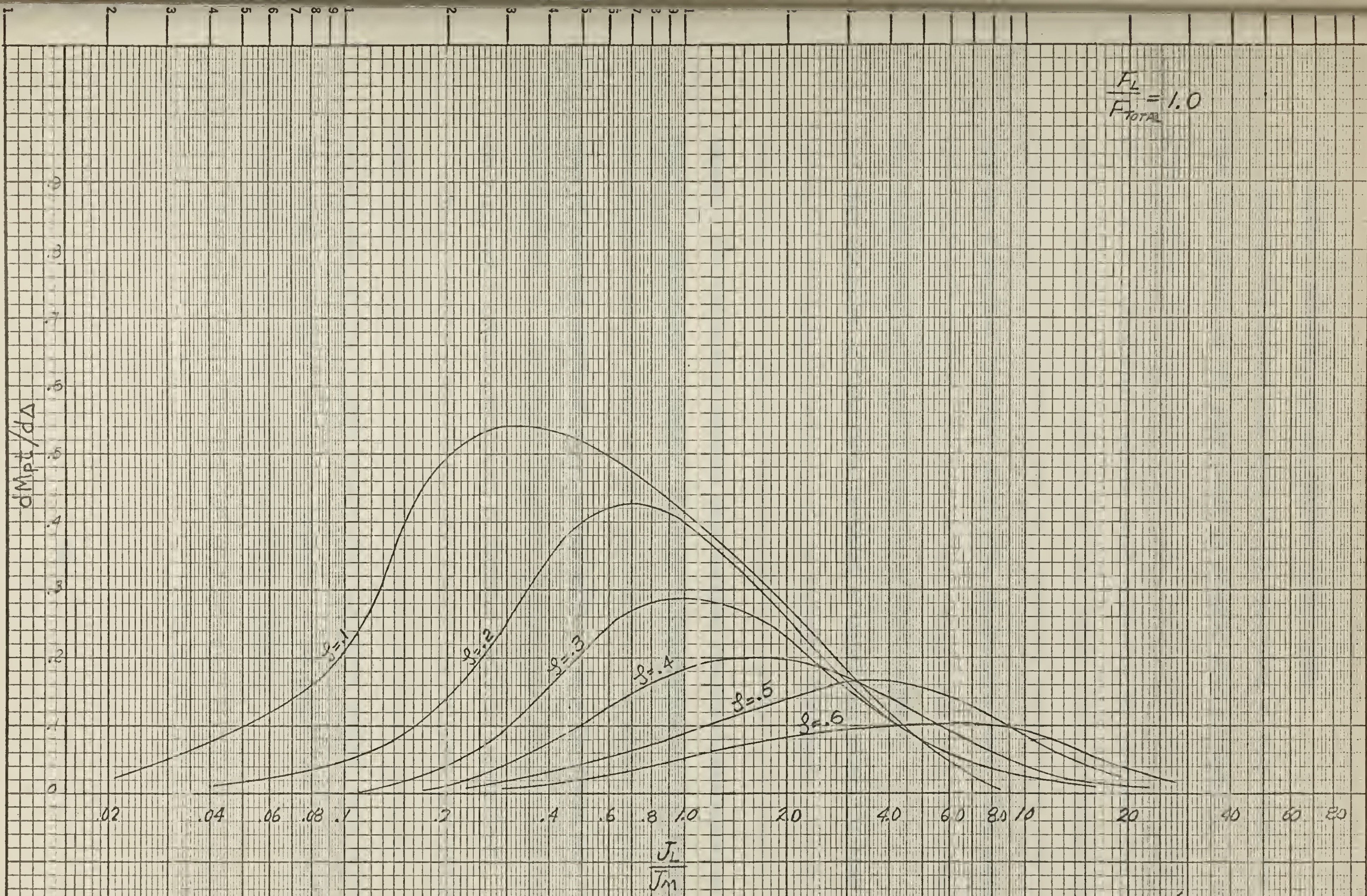


FIG. 21 DESIGN CHART FOR PREDICTING MAXIMUM OVERSHOOT $\left(\frac{F_L}{F_{TOTAL}} = 1.0\right)$

B. Maximum Overshoot for the Elastic Impact Case.

One of the assumptions made in the beginning of this study was that the contact of the gear teeth is plastic with no bouncing of the gears. Since this study was begun, an investigation has been completed at the U. S. Postgraduate School which considers the backlash case with varying amounts of resilience in the gear teeth. This investigation, carried on by T. W. Luckett, and N. O. Anderson has produced some results bearing on maximum overshoot which will be included here. The information, while not included as part of their study, was available from the data taken from the computer solution of their problem.

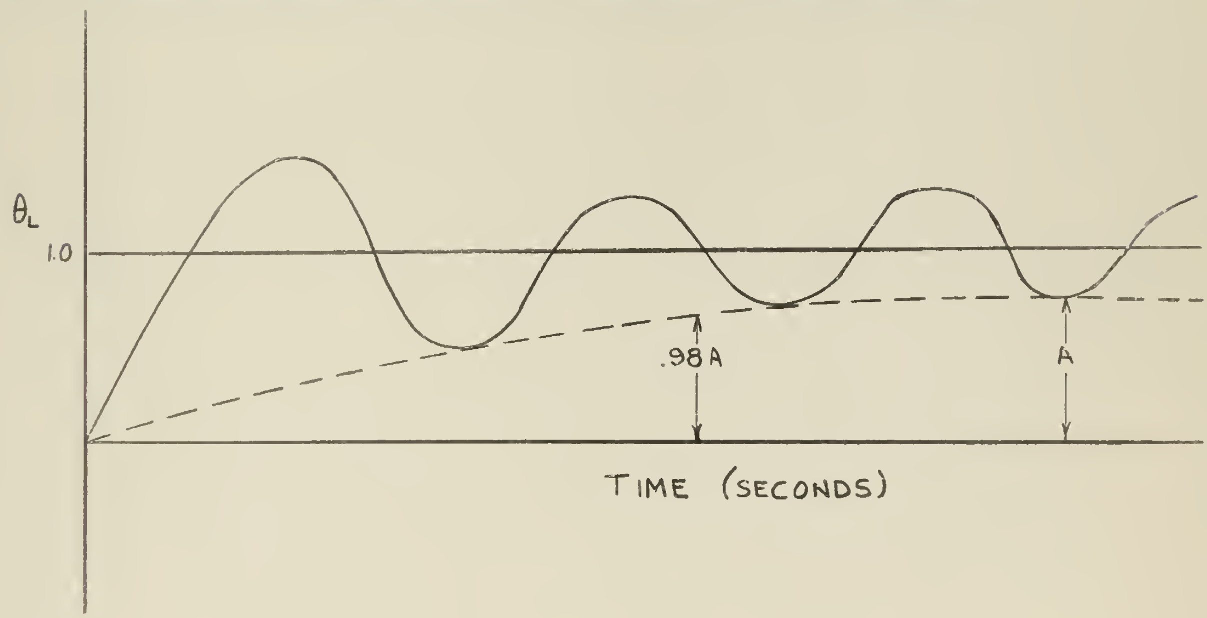
The analysis of this data revealed a surprising effect of gear bounce on the maximum overshoot of the system. The amount of overshoot with plastic impact is always greater than that with elastic impact. The difference between this maximum overshoot decreases as F_L/F_{total} increases. However, under no circumstances was the effect on maximum overshoot greater than .01 radians. The effect on the time of maximum overshoot appears negligible. In a few cases the time was different for the plastic and the elastic cases but the difference was very small and no pattern of difference could be ascertained.

These results point out the validity of the original assumption made for this study. In a practical system there will exist, of course, some amount of gear bounce when the gears contact each other. The effect on maximum overshoot, however, is so small that the curves obtained in this study remain reasonably accurate.

C. Settling Time and Time of Maximum Overshoot.

1. Settling Time

In determining the effect of backlash on the settling time of a second order servo system, it was necessary to first find a definition for settling time which would be suitable for all of the cases which were investigated. Normally, settling time is defined as the time for the output to reach 98% of its final value. This definition has real meaning, however, only for linear stable systems in which there are no sustained oscillations. For systems which do exibit small sustained oscillations, the settling time is sometimes defined as the time required for the amplitude of oscillations to decrease to within 2% of the final value. For very small amplitudes of oscillations this definition gives an unrealistic value for settling time. For the purpose of this study settling time is considered as the time required for the system output to settle to 98% of the difference between the magnitude of the input and the amplitude of the sustained oscillations as shown in the sketch below.



For all cases for which the damping coefficient was less than 0.6 there were no conclusions as to effects of backlash on the system settling time since the magnitudes of the sustained oscillations were always quite large for the amount of backlash used in the solutions and the settling time was difficult to obtain accurately. For those cases for which the damping coefficient was greater than 0.6, the sustained oscillations were small if they existed and an analysis revealed the following general conclusions:

1. As the size of the backlash angle increases, the settling time will increase provided the other parameters are kept constant. For the cases investigated, this increase in settling time was nearly proportional to the backlash angle.

2. As the ratio J_L/J_M increases, with other parameters constant, the settling time increases.

3. At the ratio F_L/F_{total} is increased, with other parameters constant, the settling time decreases.

The data included in Tables I-III show these general conclusions and also some quantitative effects of backlash on settling time.

TABLE I

SETTLING TIME DATA

ζ	J_L/J_M	F_L/F_{total}	Radians Δ	Seconds t_s
.6	1/9	.2	.3	28
.6	1/9	.4	.3	21
.6	1/9	.6	.3	16
.6	1/9	.8	.3	13
.6	1/9	1.0	.3	7
.6	1/4	.2	.3	28
.6	1/4	.4	.3	24
.6	1/4	.6	.3	16
.6	1/4	.8	.3	14
.6	1/4	1.0	.3	9
.6	1	.2	.3	29
.6	1	.4	.3	28
.6	1	.6	.3	23
.6	1	.8	.3	17
.6	1	1.0	.3	10
.6	4	.2	.3	29
.6	4	.4	.3	28
.6	4	.6	.3	26
.6	4	.8	.3	23
.6	4	1.0	.3	10
.6	9	.2	.3	29
.6	9	.4	.3	29
.6	9	.6	.3	26
.6	9	.8	.3	23
.6	9	1.0	.3	10

TABLE II

SETTLING TIME DATA

ζ	J_L/J_M	F_L/F_{total}	Radians Δ	Seconds t_s
.6	1/9	.8	.03	5
.6	1/9	.8	.10	7
.6	1/9	.8	.15	8
.6	1/9	.8	.30	13
.6	1	.8	.03	11
.6	1	.8	.10	14
.6	1	.8	.15	16
.6	1	.8	.30	17
.6	4	.8	.03	12
.6	4	.8	.10	14
.6	4	.8	.15	17
.6	4	.8	.30	23
.6	9	.8	.03	13
.6	9	.8	.10	16
.6	9	.8	.15	18
.6	9	.8	.30	23

TABLE III

SETTLING TIME DATA

ζ	J_L/J_M	F_L/F_{total}	Radians Δ	Seconds t_s
.8	1/9	.2	.3	8
.8	1/9	.4	.3	7
.8	1/9	.6	.3	6
.8	1/9	.8	.3	5
.8	1/9	1.0	.3	5
.8	1/4	.2	.3	20
.8	1/4	.4	.3	9
.8	1/4	.6	.3	7
.8	1/4	.8	.3	6
.8	1/4	1.0	.3	5
.8	1	.2	.3	43
.8	1	.4	.3	31
.8	1	.6	.3	10
.8	1	.8	.3	8
.8	1	1.0	.3	6
.8	4	.2	.3	48
.8	4	.4	.3	36
.8	4	.6	.3	12
.8	4	.8	.3	9
.8	4	1.0	.3	6
.8	9	.2	.3	52
.8	9	.4	.3	43
.8	9	.6	.3	13
.8	9	.8	.3	9
.8	9	1.0	.3	6

2. Time of Maximum Overshoot

For each of the cases investigated the time of maximum overshoot (peak time) was recorded from the printout of the solution. It was found that the presence of backlash causes a slight increase in the peak time over the linear system for those cases where $\zeta > .5$, and has negligible effect in those where $\zeta < .5$. When F_L/F_{total} is increased from zero to 0.5, other parameters constant and $\zeta > .5$, peak time increases slightly and remains essentially constant for $F_L/F_{total} > .5$. As J_L/J_M is increased, other parameters constant and $\zeta > .5$, peak time is decreased.

D. Other System Characteristics.

A careful study of the results as plotted in figures 10-15 reveals some interesting phenomena. At all friction ratios it is observed that as the ratio of load inertia to motor inertia is made smaller the amplitude of the overshoot is less. There is a point where the overshoot is identical to that of a linear second order system, in other words, where backlash has no effect on the overshoot. Note that this point occurs at different inertia ratios for each value of damping ratio. Also this point varies depending on the value of friction ratio. If the inertia ratio is decreased beyond this point then the amplitude of overshoot remains constant at the value of the linear case. Figure 22 is a plot of the boundaries of the combinations of variables which will cause an effect on the amplitude of overshoot if the system has any appreciable amount of backlash. The coordinates are friction ratio and inertia ratio with the boundaries labeled with values of damping ratio. Areas to the right of a boundary are combinations of the variables which affect overshoot. These curves were plotted using information deducted from figures 10-15. Three points were selected as test points to prove the validity of the boundaries as plotted. These points are shown on the figure as points "A", "B", and "C". The system parameters used at the test points were:

Point A		Point B		Point C	
ζ	= 0.8	ζ	= 0.6	ζ	= 0.3
F_L/F_{total}	= 0.8	F_L/F_{total}	= 0.6	F_L/F_{total}	= 0.4
J_L/J_M	= 1.0	J_L/J_M	= 0.25	J_L/J_M	= 0.111

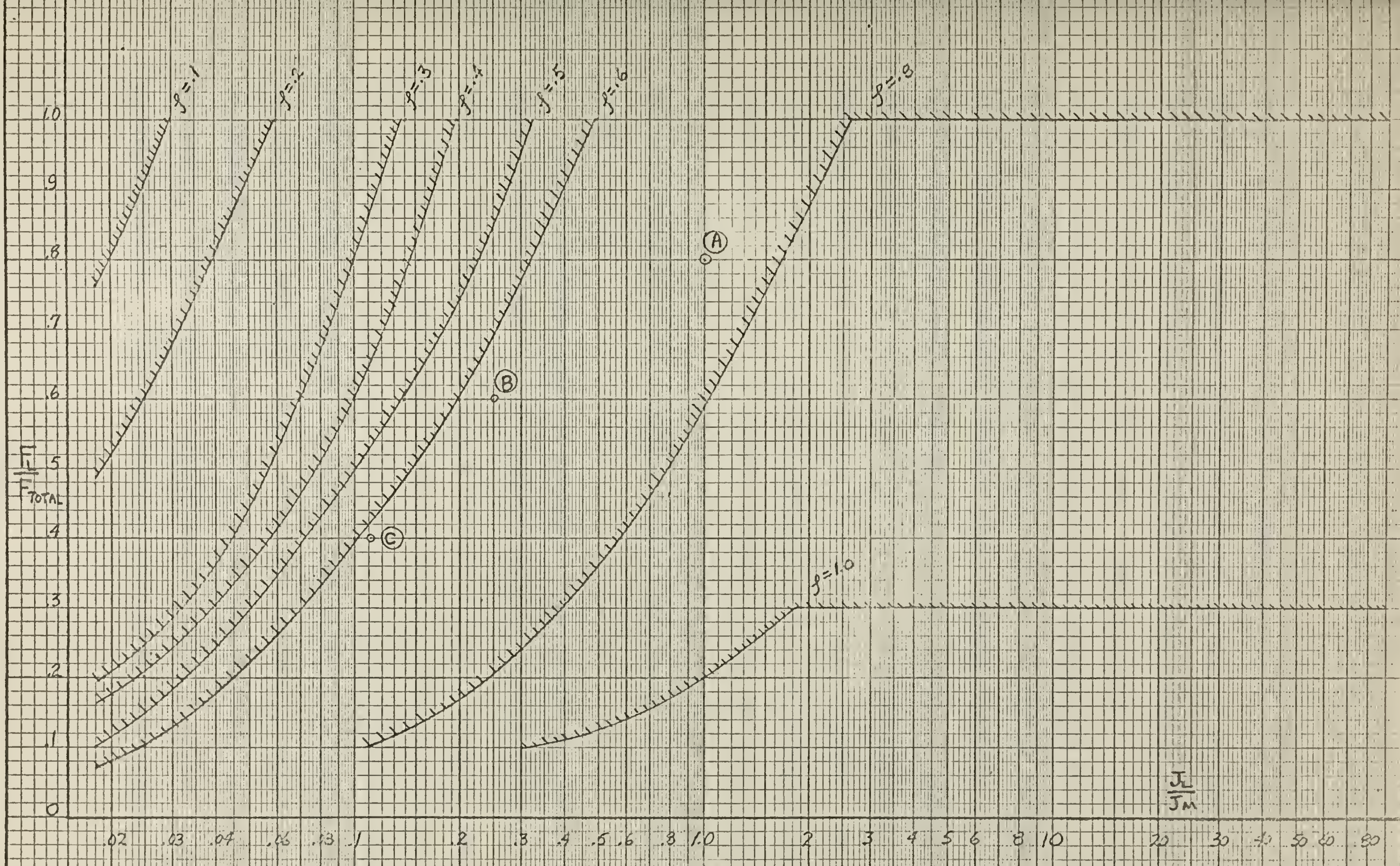


FIG. 22 REGIONS OF PARAMETER COMBINATIONS FOR WHICH BACKLASH AFFECTS MAXIMUM OVERSHOOT

The system was solved on the digital computer to obtain the maximum overshoot for these three sets of parameters. According to the curves point A falls to the left of the boundary of the $\int$ = .8 area. Therefore, the overshoot should be the same as that of the linear case. The computer solution gave an overshoot of 1.019, which is, in fact, identical to that of the linear case as seen in figure 9. Similarly, point B falls to the right of the $\int$ = 0.6 boundary and in the area where the amplitude of overshoot should not be the same as for the linear case. The computer solution gave a maximum overshoot of 1.110 which is slightly greater than the linear case. Point C falls to the right of the $\int$= 0.3 boundary and in the area where the amplitude of overshoot should be different from that of the linear system. The computer solution gave a maximum overshoot of 1.41 while that of the linear system is 1.372. From these boundary curves one cannot determine the exact effect of backlash on the maximum overshoot of a system, but can determine in which systems backlash will cause an appreciable effect.

Another phenomena which is revealed in the results as plotted in figures 10-15 is that for all cases the greatest maximum overshoot occurs in the range $0.1 < J_L/J_M < 10.0$ for the range of J_L/J_M tested. However, it appears that since the curves are trending upwards at the high end of the J_L/J_M axis that at very large inertia ratios the overshoot would be greater. It is felt that these large inertia ratios are outside the practical range. An attempt was made to explain these phenomena but without success. It is often very difficult, if not impossible, to find a physical explanation for certain behavior caused by non-linearities. They can merely be pointed out from analyses of

experimental information and accounted for accordingly.

It is interesting to note that when the ratio of load friction to motor friction is below 0.4, as in figures 10 and 11, there is a possibility of an overshoot for the critically damped case, i.e. $\zeta = 1.0$, depending on the inertia ratio. At first glance one would reason that if the system is critically damped there should be no overshoot. However, it can be seen that if the friction ratio is small and the inertia ratio is large then the ratio of F_L/J_L, which is the slope of the drifting load trajectory, is made small. Therefore, when the load separates from the motor, its velocity remains fairly constant at a relatively high value and its momentum carries it over the input to cause an overshoot.

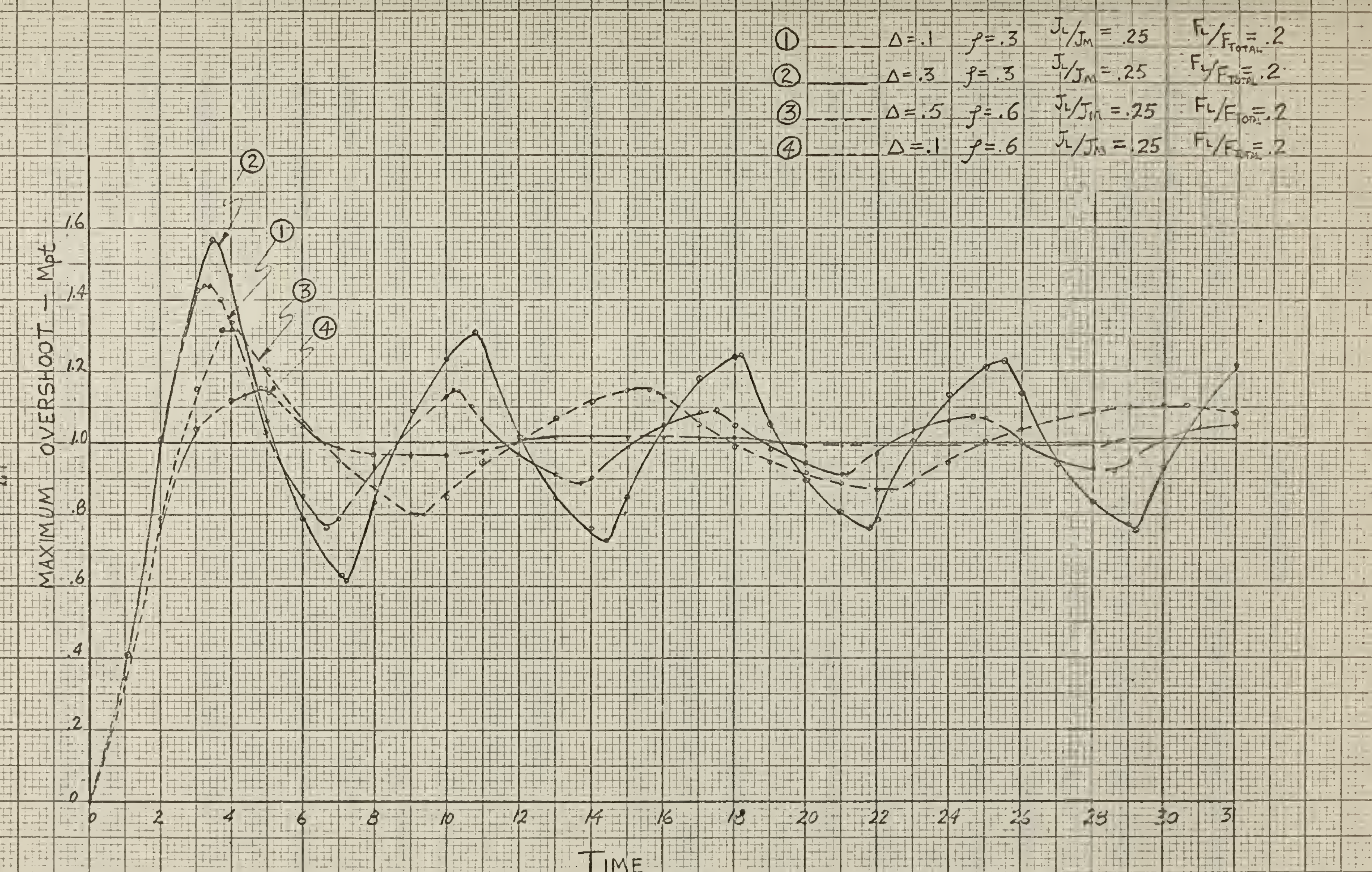


FIG. 23 WAVEFORMS OF TRANSIENT OSCILLATIONS IN RESPONSE TO A UNIT STEP INPUT WHEN BACKLASH IS PRESENT

Chapter VI

CONCLUSIONS

The following is a summary of facts which can be concluded from the results of this investigation:

1. The amplitude of maximum overshoot in a second order servo system with backlash bears a linear relationship with the magnitude of the backlash angle. The slope of the straight line relating overshoot to the backlash angle varies depending on the division of inertia between motor and load, the division of friction between motor and load, and the system damping coefficient. The design charts included in this thesis, drawn from the results of many solutions of the equations of the motion of the system and based on a backlash angle of 0.3 radian, can be used to predict the amount of overshoot of a particular servo system.

2. The fact that the gear teeth have a certain amount of resilience and the contact is not plastic, has negligible effect on the results obtained in this study.

3. The primary effect of backlash on the settling time of a second order servo is to increase it, the increase being proportional to the magnitude of the backlash angle. Due to the presence of limit cycles of sizable magnitudes for systems with $\zeta < .6$, settling time for those systems was difficult to define and no conclusions were reached. For those systems with $\zeta > .6$: (a) Increasing F_L/F_{total} decreases settling time, (b) Increasing J_L/J_M increases settling time, (c) Increasing the damping coefficient increases settling time, and (d) Increasing backlash increases settling time.

4. The presence of backlash in a system causes a slight increase in the time of maximum overshoot (peak time) for those systems where $\zeta > .5$,

and has negligible effect in those where $\zeta < .5$. When F_L/F_{total} is increased from zero to 0.5, other parameters constant and $\zeta > .5$, peak time increases slightly and remains essentially constant for $F_L/F_{total} > .5$. As J_L/J_M is increased, other parameters constant and $\zeta > .5$, peak time is decreased.

5. With certain combinations of parameters present in a system, backlash effects no change in the amplitude of overshoot over that of the linear system. Regions of these combinations are presented in chart form in figure 22.

6. For all friction ratios and damping coefficients the greatest maximum overshoot, when backlash is present, occurs in the range $0.1 < J_L/J_M < 10.0$ for the range of J_L/J_M investigated.

7. In systems where $F_L/F_{total} < .4$, when backlash is present, there will be an overshoot even for critically damped systems if J_L/J_M is high enough.

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APPENDIX A

USE OF DESIGN CHARTS

In order to demonstrate the procedure for using the design charts shown in figures 10-21, an example problem is solved obtaining the maximum overshoot, M_{pt}, for a second order servo with backlash. From Ref. 2, the differential equation for the combined motion of an armature controlled motor is:

$$\ddot{\theta} + \frac{F_T + F_l}{J_M + J_L}\dot{\theta} + \frac{K_a}{J_M + J_L}\theta = \frac{K_a}{J_M + J_L}\theta_R \quad \text{(A-1)}$$

Given the following specifications of a D. C. shunt motor coupled through a gear box to a load:

Horsepower	= .008
Rated RPM	= 4000
R	= 282 ohms
K_g	= .233 volts/radian/second
K_t	= .1464 lb.-feet/ampere
J_m	= 8.2×10^{-6} slug-feet^2
F_m	= 3.34×10^{-6} lb.-feet/radian/sec.
K_e	= 10 volts/radian
ρ	= 1
Δ	= .05
J_L	= 16.4×10^{-6} slug-feet^2
F_L	= 10.4×10^{-6} lb.-feet/radian/sec.

The required values shown in Equation A-1 are obtained as follows:

$$K_a = \frac{K_t K_e}{\rho R} = \frac{.1464 \times 10}{(1)\times 282} = \underline{.0052} \text{ LB-FT}$$

$$J_M = \frac{J_m}{\rho^2} = \frac{8.2 \times 10^{-6}}{(1)^2} = \underline{8.2} \text{ SLUG-FT}$$

$$\omega_n = \sqrt{\frac{K_a}{J_M + J_L}} = \sqrt{\frac{.0052}{8.2\times 10^{-6} + 16.4 \times 10^{-6}}} = \underline{14.5} \frac{\text{RAD}}{\text{SEC}}$$

$$F_T = F_L + F_m/\rho^2 = 10.4 \times 10^{-6} + \frac{3.34 \times 10^{-6}}{(1)^2}$$

$$= \underline{13.74 \times 10^{-6}} \frac{\text{LB-FT}}{\text{RAD/SEC}}$$

$$F_{total} = F_T + F_1 = (13.74 + 121) \times 10^{-6} = \underline{135 \times 10^{-6}} \frac{\text{LB-FT}}{\text{RAD/SEC}}$$

$$F_1 = \frac{K_t K_g}{\rho^2 R} = \frac{.1464 \times .233}{(1)^2 \times 282} = \underline{121 \times 10^{-6}} \frac{\text{LB-FT}}{\text{RAD/SEC}}$$

$$\zeta = \frac{F_T + F_1}{J_M + J_L}\left(\frac{1}{2\omega_n}\right) = \underline{.19}$$

$$J_L/J_M = \frac{16.4 \times 10^{-6}}{8.2 \times 10^{-6}} = \underline{2.0}$$

$$F_L/F_{total} = \frac{10.4 \times 10^{-6}}{135 \times 10^{-6}} = \underline{.077}$$

Two methods are shown for obtaining the maximum overshoot with a backlash angle of .05 radians in the gear trains.

The first method is accomplished using the curves of figures 10-15. The procedure is simple, but requires more steps than does the second. Also one must remember that these curves are drawn for a backlash of 0.3 radian and must be scaled for the expected amount of backlash. The second method makes use of the design charts of figures 16-21.

Method 1

Entering figure 10, ($F_L/F_{total} = 0.1$), at $J_L/J_M = 2.0$, and a damping ratio, $\zeta = 0.19$, the amount of overshoot for a backlash angle, $\Delta =$ 0.3 rad. is $M_{pt} = 1.658$.

Using Equation V-1

$$M_{pt}\Big|_{\substack{\Delta=.05\\ \zeta=.19}} = M_{pt}\Big|_{\substack{\Delta=0\\ \zeta=.19}} + .05/.3 \left[M_{pt}\Big|_{\substack{\Delta=.3\\ \zeta=.19}} - M_{pt}\Big|_{\substack{\Delta=0\\ \zeta=.19}} \right]$$

From the second order linear characteristics curve of figure 9

$$M_{pt} = 1.545$$

hence,

$$M_{pt} = 1.545 + .05/.3 \quad 1.658 - 1.545$$

$$= 1.545 + .0189$$

$$= \underline{1.564}$$

Method 2

Using Equation V-3

$$M_{pt}\Big|_{\substack{\Delta=.05\\ \zeta=.19}} = M_{pt}\Big|_{\substack{\Delta=0\\ \zeta=.19}} + dM_{pt}/d\Delta\Big|_{\zeta=.19} \times \Delta$$

from figure 16, ($F_L/F_{total} = 0.1$), at $J_L/J_M = 2.0$, and a damping ratio, $= 0.19$, we obtain $dM_{pt}/d\Delta = 0.357$

From figure 9

$$M_{pt}\Big|_{\substack{\Delta=0\\ \zeta=.19}} = 1.545$$

hence,

$$M_{pt} = 1.545 + .357 \times .05$$

$$= 1.545 + .018$$

$$= \underline{1.563}$$

This compares closely to the answer obtained using Method 1.

The procedure for using the charts remains unchanged for servo systems which utilize other types of motors. The only difference which arises is in the calculation of the chart parameters from the known system constants. For a field control motor if the field time constant is negligible, the equation is similar to the armature control motor equation.

$$\ddot{\theta} + \frac{F_T}{J_M + J_L}\dot{\theta} + \frac{K_a}{J_M + J_L}\theta = \frac{K_a}{J_M + J_L}\theta_R \qquad \text{(A-2)}$$

In like manner the equation for the system with a two phase motor is

$$\ddot{\theta} + \frac{F_T + K_n}{J_M + J_L}\dot{\theta} + \frac{K_m}{J_M + J_L}\theta = \frac{K_m}{J_M + J_L}\theta_R \qquad \text{(A-3)}$$

where

$K_n = -\dfrac{\partial T}{\partial n}$ (from characteristic curves of two phase motor)

$K_m = \dfrac{\partial T}{\partial e_1}$ (from characteristic curves of two phase motor)

The calculation of the chart parameters is a simple matter once the defining equations are known.

APPENDIX B

COMPUTER PROGRAM

```
                                              REM  BACKLASH PROBLEM PLASTIC
                                                    CONTACT RUN 3
                                              ORG  20000
20000  75 4 20022   START      SLJ    4    INPUT 1      REMARKS IN THIS
       00 0 00000                0    0    0             COLUMN
20001  75 4 20063   COMBINED   SLJ    4    RESET
       00 0 00000                0    0    0
20002  75 4 60200              SLJ    4    RUNGE        SET UP GILL
       50 0 00000              ENI    0    0             (Procedure to solve
20003  00 0 20023                0    0    TABLE 1       differential
       00 0 20034                0    0    DERC          equations)
20004  75 4 60201   POINTC     SLJ    4    RUNGE + 1
       00 0 00000                0    0    0
20005  75 4 60200   LOAD 1     SLJ    4    RUNGE        SET UP GILL
       50 0 00000              ENI    0    0
20006  00 0 20040                0    0    TABLE L
       00 0 20051                0    0    DERL
20007  75 4 60201   POINTL     SLJ    4    RUNGE + 1
       00 0 00000                0    0    0
20010  75 4 20130              SLJ    4    COUNT
       50 0 00000              ENI    0    0
20011  75 0 20061              SLJ    0    TEST 1
       50 0 00000              ENI    0    0
20012  75 4 60200   MOTOR      SLJ    4    RUNGE        SET UP GILL
       50 0 00000              ENI    0    0
20013  00 0 20023                0    0    TABLE 1
       00 0 20054                0    0    DERM
20014  75 4 60201   POINTM     SLJ    4    RUNGE + 1
       00 0 00000                0    0    0
20015  75 4 60200   LOAD2      SLJ    4    RUNGE        SET UP GILL
       50 0 00000              ENI    0    0
20016  00 0 20040                0    0    TABLE L
       00 0 20051                0    0    DERL
20017  75 4 60201   POINT2L    SLJ    4    RUNGE + 1
       000 0 00000               0    0    0
20020  75 4 20130              SLJ    4    COUNT
       50 0 00000              ENI    0    0
20021  75 0 20073              SLJ    0    TEST 2
       00 0 00000                0    0    0
20022  75 0 00000   INPUT1     SLJ    0    0            Provision for chang-
       75 0 20022              SLJ    0    INPUT1      ing Program if desired
20023  00 0 00000   TABLE1     DEC         2           Table for gill combin-
       00 0 00002                                      ed
20024  17 7 15075              DEC         .01
       34 1 21727
20025  00 0 00000   T          DEC         0
       00 0 00000
```

```
20026   00 0 00000   UDOT       DEC        0
        00 0 00000
20027   00 0 00000   U          DEC        0
        00 0 00000
20030   00 0 00000   QU         DEC        0
        00 0 00000
20031   00 0 00000   THETADOT   DEC        0
        00 0 00000
20032   00 0 00000   THETA      DEC        0
        00 0 00000
20033   00 0 00000   QTHETA     DEC        0
        00 0 00000
20034   13 0 20156   DERC       LAC    0   B              Derivative for com=
                                                           bined
        32 0 20027              FMU    0   U
20035   31 0 20032              FSB    0   THETA
        30 0 20164              FAD    0   ONE
20036   20 0 20026              STA    0   UDOT
        12 0 20027              LDA    0   U
20037   20 0 20031              STA    0   THETADOT
        75 0 60202              SLJ    0   RUNGE + 2
20040   00 0 00000   TABLEL     DEC        2              Table for gill load
        00 0 00002
20041   17 7 15075              DEC        .01
        34 1 21727
20042   00 0 00000   TL         DEC        0
        00 0 00000
20043   00 0 00000   WDOT       DEC        0
        00 0 00000
20044   00 0 00000   W          DEC        0
        00 0 00000
20045   00 0 00000   QW         DEC        0
        00 0 00000
20046   00 0 00000   THETALD    DEC        0
        00 0 00000
20047   00 0 00000   THETAL     DEC        0
        00 0 00000
20050   00 0 00000   QTHETAL    DEC        0
        00 0 00000
20051   13 0 20157   DERL       LAC    0   D              Derivative for load
        32 0 20044              FMU    0   W
20052   20 0 20043              STA    0   WDOT
        12 0 20044              LDA    0   W
20053   20 0 20046              STA    0   THETALD
        75 0 60202              SLJ    0   RUNGE + 2
20054   13 0 20161   DERM       LAC    0   F              Derivative for motor
        32 0 20027              FMU    0   U
20055   31 0 20047              FSB    0   THETAL
        30 0 20164              FAD    0   ONE
20056   33 0 20160              FDV    0   E
        20 0 20026              STA    0   UDOT
20057   12 0 20027              LDA    0   U
        20 0 20031              STA    0   THETADOT
```

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20060   75 0 60202              SLJ   0   RUNGE + 2
        50 0 00000              ENI   0   0
20061   12 0 20046   TEST1      LDA   0   THETHALD    Test to determine
        31 0 20031              FSB   0   THETADOT     Start of backlash
20062   22 2 20012   JUMP1      AJP   2   MOTOR
        75 0 20001              SLJ   0   COMBINED
20063   75 0 00000   RESET      SLJ   0   0
        12 0 20025              LDA   0   T           Changes load initial
20064   20 0 20042              STA   0   TL           conditions back to
        12 0 20026              LDA   0   UDOT         motor initial con-
20065   20 0 20043              STA   0   WDOT         ditions if separa-
        12 0 20027              LDA   0   U            tion has not occurr-
                                                       ed
20066   20 0 20044              STA   0   W
        12 0 20031              LDA   0   THETADOT
20067   20 0 20046              STA   0   THETALD
        12 0 20032              LDA   0   THETA
20070   20 0 20047              STA   0   THETAL
        10 0 00000              ENA   0   0
20071   20 0 20045              STA   0   QW
        20 0 20050              STA   0   QTHETAL
20072   75 0 20063              SLJ   0   RESET
        50 0 00000              ENI   0   0
20073   12 0 20047   TEST2      LDA   0   THETAL      Test to determine if
        31 0 20032              FSB   0   THETA        backlash has been
20074   31 0 20166   BKTEST2    FSB   0   BACKLASH     taken up
        50 0 00000              ENI   0   0
20075   22 2 20137   JUMP2      AJP   2   CONTACT
        75 0 20012              SLJ   0   MOTOR
20076   36 0 20117   SWITCH     SSK   0   ALTERNAT    Plan for changing
        75 0 20100              SLJ   0   LEFT         sign of backlash
20077   75 0 20107              SLJ   0   RIGHT        each cycle
        50 0 00000              ENI   0   0
20100   12 0 20062   LEFT       LDA   0   JUMP 1
        14 0 20116              ADD   0   FACTOR
20101   20 0 20062              STA   0   JUMP 1
        12 0 20075              LDA   0   JUMP 2
20102   14 0 20116              ADD   0   FACTOR
        20 0 20075              STA   0   JUMP 2
20103   12 0 20074              LDA   0   BKTEST2
        15 0 20172              SUB   0   FACTOR2
20104   20 0 20074              STA   0   BKTEST2
        12 0 20117              LDA   0   ALTERNAT
20105   05 0 00001              ALS   0   1
        20 0 20117              STA   0   ALTERNAT
20106   75 0 20001              SLJ   0   COMBINED
        50 0 00000              ENI   0   0
20107   12 0 20062   RIGHT      LDA   0   JUMP1
        15 0 20116              SUB   0   FACTOR
20110   20 0 20062              STA   0   JUMP1
        12 0 20075              LDA   0   JUMP2
20111   15 0 20116              SUB   0   FACTOR
        20 0 20075              STA   0   JUMP2
20112   12 0 20074              LDA   0   BKTEST2
```

```
        14 0 20172              ADD   0  FACTOR2
20113   20 0 20074              STA   0  BKTEST2
        12 0 20117              LDA   0  ALTERNAT
20114   05 0 00001              ALS   0  1
        20 0 20117              STA   0  ALTERNAT
20115   75 0 20001              SLJ   0  COMBINED
        50 0 00000              ENI   0  0
20116   00 1 00000  FACTOR      OCT      0010000000000000
        00 0 00000
20117   25 2 52525  ALTERNAT    OCT      2525252525252525
        25 2 52525
20120   75 0 00000  PRINT       SLJ   0  0          Print routine
        12 0 20025              LDA   0  T
20121   20 0 20252              STA   0  BUF
        12 0 20027              LDA   0  U          Motor speed
20122   20 0 20253              STA   0  BUF + 1
        12 0 20032              LDA   0  THETA      Motor position
20123   20 0 20254              STA   0  BUF + 2
        12 0 20044              LDA   0  W          Load speed
20124   20 0 20255              STA   0  BUF + 3
        12 0 20047              LDA   0  THETAL     Load position
20125   20 0 20256              STA   0  Buf + 4
        75 4 71000              SLJ   4  DECO
20126   01 0 20252               01   0  BUF
        06 0 00001               06   0  1
20127   72 0 20126              RAO   0  /-1
        75 0 20120              SLJ   0  PRINT
20130   75 0 00000  COUNT       SLJ   0  0
        12 0 20170              LDA   0  INDEX      Plan for printing
20131   14 0 20171              ADD   0  INCRONE     only every tenth
        15 0 20173              SUB   0  TEN         point at .08 sec
20132   22 0 20134              AJP   0  CK2PRINT    interval
        14 0 20173              ADD   0  TEN
20133   20 0 20170              STA   0  INDEX
        75 0 20130              SLJ   0  COUNT
20134   75 4 20120  OK2PRINT    SLJ   4  PRINT
        50 0 00000              ENI   0  0
20135   10 0 00000              ENA   0  0
        20 0 20170              STA   0  INDEX
20136   75 0 20130              SLJ   0  COUNT
        50 0 00000              ENI   0  0
20137   12 0 20162  CONTACT     LDA   0  G          Apply law of
        32 0 20027              FMU   0  U           momentum
20140   20 0 20165              STA   0  GU
        12 0 20163              LDA   0  H
20141   32 0 20044              FMU   0  W
        30 0 20165              FAD   0  GU
20142   20 0 20031              STA   0  THETADOT   Puts resulting values
        20 0 20027              STA   0  U           of variables in
20143   20 0 20044              STA   0  W           Table 1 and L
        20 0 20046              STA   0  THETALD
20144   13 0 20156              LAC   0  B
        32 0 20027              FMU   0  U
```

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20145   31 0 20032                 FSB     0   THETA
        30 0 20164                 FAD     0   ONE
20146   20 0 20026                 STA     0   UDOT
        13 0 20157                 LAC     0   D
20147   32 0 20044                 FMU     0   W
        20 0 20043                 STA     0   WDOT
20150   12 0 20047                 LDA     0   THETAL
        20 0 20032                 STA     0   THETA
20151   10 0 00000                 ENA     0   0
        20 0 20030                 STA     0   QU
20152   20 0 20033                 STA     0   QTHETA
        20 0 20045                 STA     0   QW
20153   20 0 20050                 STA     0   QTHETAL
        75 4 20120                 SLJ     4   PRINT
20154   50 0 00000                 ENI     0   0
        75 0 20076                 SLJ     0   SWITCH
20155   50 0 00000                 ENI     0   0
        50 0 00000
20156   17 7 66314   B             DEC         .4              B equals twice the
        63 1 46314                                              damping coefficient
20157   17 7 54000   D             DEC         .125            D equals the load
        00 0 00000                                              time constant
20160   17 7 56314   E             DEC         .2              E equals motor iner-
        63 1 46314                                              tia
20161   17 7 64631   F             DEC         .3              F equals motor fric-
        46 3 14631                                              tion
20162   17 7 56314   G             DEC         .2              G equals motor iner-
        63 1 46314                                              tia
20163   20 0 06314   H             DEC         .8              H equals load iner-
        63 1 46314                                              tia
20164   20 0 14000   ONE           DEC         1.              Integer
        00 0 00000
20165   00 0 00000   GU            DEC         0               Constant initially
        00 0 00000                                              zero
20166   17 7 64631   BACKLASH      DEC         .3              Amount of backlash
        46 3 14631
20167   01 0 00000   FACTOR2       OCT     0100000000000000
        00 0 00000
20170   00 0 00000   INDEX         OCT         0
        00 0 00000
20171   00 0 00000   INCRONE       OCT         1               Used in count
        00 0 00001
20172   00 0 00000   BLANK         OCT         0
        00 0 00000
20173   00 0 00000   TEN           OCT         10              Used in count
        00 0 00010
20174   17 7 15075   1             DEC         .01             Beginning of float-
        34 1 21727                                              ing point conver-
                                                                sion table
```